Computability and Complexity

Instituto de Matemáticas

Exam

Regular and context-free languages

Guidelines: read before anything!

This exam is divided in three sections, and will give you two grades: one for regular languages and one for context-free languages.

1. All exercises in Section 1 will count towards your grade for regular languages;
2. All exercises in Section 3 will count towards your grade for context-free languages;
3. The exercise in Section 2 will count towards both grades.

You can do any exercises as long as you are sure to get a passing grade for each part.

1 Regular languages

Exercise 1 : ***Warmup (3 points).***
For each of the following languages, construct a finite automata that recognizes them:

1. $\{w \in \{a, b\}^* \mid w \text{ contains an even number of } a \text{ and an odd number of } b\}$;
2. $\{w \in \{a, b\}^* \mid w \text{ does not contain } bab\}$.

□

Exercise 2 : ***Opening by closing (2 points).***
In the next questions, assume that all languages are over a common alphabet Σ.

1. Prove that the class of regular languages is closed under difference (*i.e.*, if L_1 and L_2 are regular, then $L_1 \setminus L_2$ is regular).
2. Deduce from the previous result that the class of regular languages is closed under complementation.

□

Exercise 3 : ***Finition (2 points).***
Prove that if L is a finite language, then L is regular.

□

Exercise 4 : ***Mr. Pump (2 points).***
Prove that the following languages are not regular:

1. $\{ww \mid w \in \{a, b\}^*\}$;
2. $\{a^{2^n} \mid n \geq 0\}$ (the chains of 2^n consecutive a's for any nonnegative integer n).

□

2 Transition

Exercise 5 : ***An irregular with no context (3 points).***
Let $L = \{a^m b^n \mid m \neq n;\ m, n \geq 0\}$.

1. Prove that L is not regular.
2. Prove that L is context-free by constructing a pushdown automata that recognizes it.

$\square$

3 Context-free languages

Exercise 6 : ***Elementary grammars (2 points).***
For each of the following languages, give a context-free grammar that generates it:

1. $\{a^m b^{m+n} a^n \mid m, n \geq 0\}$;
2. $\{w \in \{a, b\}^* \mid w$ contains more a's than b's$\}$;
3. The complement of $\{a^n b^n \mid n \geq 0\}$.

$\square$

Exercise 7 : ***Regulars don't have context (3 points).***
Prove that every regular language is context-free, by showing how to convert a regular expression to an equivalent context-free grammar.

$\square$

Exercise 8 : ***Pumping in context (2 points).***
Prove that the language $\{ww \mid w \in \{a, b\}^*\}$ is not context-free.

$\square$

Exercise 9 : ***Challenging grammars (4 points).***
For each of the following languages, give a context-free grammar that generates it:

1. $\{w \in \{a, b\}^* \mid w = xy$ with $|x| = |y|$ but $x \neq y\}$;
2. $\{w\#x \mid w^R$ is a substring of $x;\ w, x \in \{a, b\}^*\}$.

$\square$