Computability and Complexity

Exercises

Finite automatas and regular languages

Exercise 1 : *First automatas*.
Let $\Sigma = \{a, b\}$. Construst a deterministic automata that accepts the following:

1. No word over Σ;
2. Any word over Σ;
3. Only the words composed of one symbol of Σ;
4. $a^* \cup b^*$.

□

Exercise 2 : *Salad of automatas*.
Let $\Sigma = \{a, b\}$. Construct an automata that accepts the following languages:

1. $L_1 = \{w \in \Sigma^* \mid \text{every } a \text{ in } w \text{ is immediately preceded and followed by a } b\}$;
2. $L_2 = \{w \in \Sigma^* \mid ab \notin w \text{ and } ba \notin w\}$;
3. $L_3 = \{w \in \Sigma^* \mid ab \in w \text{ or } ba \in w\}$;
4. $L_4 = \{w \in \Sigma^* \mid ab \in w \text{ and } ba \in w\}$;
5. $L_5 = \{w \in \Sigma^* \mid w \text{ contains an odd number of } a \text{ and an odd number of } b\}$;
6. $L_6 = \{w \in \Sigma^* \mid aaa \notin w \text{ and } w \text{ contains an odd number of } b\}$;
7. $L_7 = \{w \in \Sigma^* \mid aa \in w \text{ and the first occurrence of } aa \text{ is not preceded by } abab\}$.

□

Exercise 3 : *Binary words*.
Let $\Sigma = \{0, 1\}$. Construct an automata that accepts the following languages:

1. $L_0 = \{0^m 1^n \mid m, n \geq 0\}$;
2. $L_1 = \{0^m 1^n \mid m, n > 0\}$;
3. $L_{10} = \{w \in \Sigma^* \mid w \text{ ends with } 101\}$;
4. $L_{11} = \{w \in \Sigma^* \mid w \equiv 0 \bmod 3\}$.

□

Exercise 4 : *Binary operations*.
Let L be a language. We define L^R as the language containing all the words in L written in reverse (for example, if L contains $abab$ and $xkcd$ then L^R contrains $baba$ and $dckx$).

1. Prove that if L is regular, then L^R is regular.

Let $\Sigma_2 = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$. Note that any word $w \in \Sigma_2^+$ can be read as two binary numbers, one for each row. The top number is denoted by w^t and the bottom number by w^b.

2. Prove that the language $L = \{w \in \Sigma_2^+ \mid w^t > w^b\}$ is regular.

Let $\Sigma_3 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \ldots, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$. Again, for every word $w \in \Sigma_3^+$ we get three binary numbers. The top number is w^t, the middle number is w^m and the bottom number is w^b.

3. Prove that the language $L = \{w \in \Sigma_3^+ \mid w^b = w^t + w^m\}$ is regular.

□

Exercise 5 : ***The Baker Street Irregulars.***
Use the pumping lemma to prove that the following languages are not regular:

1. $L_1 = \{a^n b^n \mid n \geq 0\}$;
2. $L_2 = \{a^{n^2} \mid n > 0\}$;
3. $L_3 = \{a^n b^{2n} \mid n > 0\}$;
4. $L_4 = \{(ab)^n c^n \mid n \geq 0\}$.

Use the properties of regular languages to prove that the following language is not regular:

5. $L_5 = \{a^m b^n \mid m \neq n\}$.

Prove in two different ways that the following language is not regular:

6. $L = \{w \in \{a, b\}^* \mid w = w^R\}$ (that is, L is the language of all the palindromes over $\{a, b\}$).

Hint: One method uses the pumping lemma, the other one the fact that an automata accepting L is finite.

□