

## Exercises

### Decidability

**Exercise 1 : 0 – 1 sequences.**

Prove that  $\{0, 1\}^{\mathbb{N}}$ , the set of all infinite sequences over  $\{0, 1\}$ , is uncountable.

□

**Exercise 2 : A whole language.**

Prove that  $L = \{ \langle A \rangle \mid A \text{ is a DFA and } L(A) = \Sigma^* \}$  is decidable.

□

**Exercise 3 : Regexp.**

Consider the problem of deciding whether a DFA  $A$  and a regular expression  $E$  verify  $L(A) = L(E)$ . Express this problem as a language and prove that it is decidable.

□

**Exercise 4 : Towards the infinity.**

Prove that  $L = \{ \langle A \rangle \mid A \text{ is a DFA and } L(A) \text{ is infinite} \}$  is decidable.

Hint: Think about the pumping lemma!

□

**Exercise 5 : Accepting palindromes.**

Prove that  $L = \{ \langle A \rangle \mid A \text{ is a DFA and } A \text{ accepts some palindrome} \}$  is decidable.

Hint: Think about a CFG that generates a palindrome, and prove that the intersection between a regular language and a context-free language is context-free.

□