Computability and Complexity

Exercises

Reducibility

Exercise 1 : ***Accepting the reverse.***
Prove that $L = \{< M > \mid M$ is a Turing machine and M accepts $w^R \Leftrightarrow M$ accepts $w\}$ is undecidable.

□

Exercise 2 : ***Two halts make a whole.***
Prove that $L = \{< M_1, M_2 > \mid M_1$ and M_2 are Turing machines, and $\exists w$ s.t. M_1 and M_2 accept $w\}$ is undecidable.

□

Exercise 3 : ***Rice's Theorem is not about eating.***
We say that P is a *nontrivial* property if it is neither true nor false for every computable function.
Prove the following:
Rice's Theorem *If P is a nontrivial property of the language of a Turing machine, then determining whether a given Turing machine's language has property P is undecidable.*
More formally: let P be a language consisting of Turing machine descriptions with the following properties:

1. There exist Turing machines M_1 and M_2 such that $M_1 \in P$ and $M_2 \notin P$ (*i.e.*, P is nontrivial);
2. For all Turing machines M_1 and M_2, if $L(M_1) = L(M_2)$, then $< M_1 >\in P \Leftrightarrow < M_2 >\in P$ (*i.e.*, P is a property of the Turing machines' languages).

Prove that P is undecidable.

□

Exercise 4 : ***Eating Rice.***
Use Rice's Theorem to prove that the following languages are undecidable:

1. $L_1 = \{< M > \mid M$ is a Turing machine and $L(M)$ is infinite$\}$
2. $L_2 = \{< M > \mid M$ is a Turing machine and $|L(M)| \geq 3\}$
3. $L_3 = \{< M > \mid M$ is a Turing machine and $L(M) = \Sigma^*\}$

□

Exercise 5 : ***Poisoned Rice - do not eat.***
A *useless state* in a Turing machine is a state that is never entered on any input string.
Let $L = \{< M > \mid M$ has a useless state$\}$. Prove that L is undecidable. Can you use Rice's Theorem?

□

Exercise 6 : ***Rice with kayak.***
Prove that $L = \{< M > \mid M$ is a Turing machine and $L(M)$ contains a palindrome$\}$ is undecidable, first by using Rice's Theorem and then without it.

□