Instituto de Matemáticas

Computability and Complexity

Exercises

Time complexity

1 P

Exercise 1 : ***Connections.***
Prove that $L_C = \{< G > \mid G$ is a connected undirected graph$\}$ is in P.

$\square$

Exercise 2 : ***Isoceles.***
Prove that $L_\Delta = \{< G > \mid G$ is an undirected graph containing a triangle$\}$ is in P.

$\square$

Exercise 3 : ***Modular exponentiation.***
Prove that $L_{\text{mod}} = \{< b, e, c, p > \mid b, e, c, p$ are binary integers and $b^e \equiv c \bmod p\}$ is in P.

$\square$

Exercise 4 : ***Unary is more efficient than binary?!.***
In the class, we saw that SUBSET-SUM is NP-complete. However, consider UNARY-SUBSET-SUM, an instance of SUBSET-SUM where the numbers are written in unary. Prove that UNARY-SUBSET-SUM is in P.

$\square$

2 NP-completeness

Exercise 5 : ***Double-SAT.***
Prove that D-SAT= $\{< \phi > \mid \phi$ is a boolean formula with at least two satisfying assignments$\}$ is NP-complete.

$\square$

Exercise 6 : ***Partition.***
Prove that PARTITION= $\{< A > \mid A$ is a set of integers such that there exists $A' \subseteq A$ s.t. $\sum_{a \in A'} a = \sum_{a \in A \setminus A'} a\}$ is NP-complete.

$\square$

Exercise 7 : ***Half-clique.***
Prove that HALF-CLIQUE= $\{< G > \mid G$ is an undirected graph with a complete subgraph of order $\frac{|V(G)|}{2}\}$ is NP-complete.

$\square$

Exercise 8 : ***Two cliques.***
Prove that TWO-CLIQUES= $\{< G, k > \mid G$ is an undirected graph with two disjoint cliques of order at least $k\}$ is NP-complete.

$\square$

Exercise 9 : ***Stable.***
Prove that STABLE= $\{< G, k > \mid G$ has an independent subgraph of order at least $k\}$ is NP-complete.

$\square$

Exercise 10 : ***Subgraphs.***
Prove that SUBGRAPH-ISOMORPHISM $= \{< G, H > \mid H$ is a subgraph of $G\}$ is NP-complete.

$\square$

Exercise 11 : ***0-1-matrices.***
Let M be a square matrix that has values in $\{0, 1\}$. We say that M is *simplifiable* if it is possible to transform 1's to 0's such that every row and column of M contains exactly one 1.
Prove that SIMPLIFIABLE $= \{M \mid M$ is a simplifiable matrix$\}$ is NP-complete.

$\square$

Exercise 12 : ***Domination.***
Prove that DOMINATING-SET$= \{< G, k > \mid G$ has a dominating set of size at most $k\}$ is NP-complete.

$\square$

Exercise 13 : ***$\neq$-assignments.***
For a given boolean formula in 3-CNF, a $\neq$-assignment of its variables is an assignment where, in every clause, there are at least two literals with unequal truth values. Equivalently, a $\neq$-assignment satisfies ϕ without having all three litterals set as `true` in any clause.

1. Prove that the negation of a $\neq$-assignment is also a $\neq$-assignment.
2. Let $\neq$-3-SAT be the language of all boolean formulas in 3-CNF that have a $\neq$-assignment. Find a reduction of 3-SAT to $\neq$-3-SAT.
3. Prove that $\neq$-3-SAT is NP-complete.

$\square$

Exercise 14 : ***Cutting as much as we can.***
Prove that MAX-CUT$= \{< G, k > \mid G$ has a cut of size at least $k\}$ is NP-complete (hint: reduce from $\neq$-3-SAT, G may contain multiedges).

$\square$

Exercise 15 : ***Cutting exactly as much as we need.***
Prove that EXACT-CUT$= \{< G, k > \mid G$ has a cut of size exactly $k\}$ is NP-complete.

$\square$

Exercise 16 : ***3-color.***
Prove that 3-COLOR$= \{< G > \mid G$ is an undirected graph that has a 3-colouring$\}$ is NP-complete.

$\square$

Exercise 17 : ***Clique in a restricted family.***
Prove that REG-CLIQUE $= \{< G, k > \mid G$ is a regular undirected graph with a clique of order at least $k\}$ is NP-complete.

$\square$

Exercise 18 : ***A small break.***
This is not an exercise. Give a read to the paper *Minesweeper is NP-complete*, available here:

`http://www.minesweeper.info/articles/MinesweeperIsNPComplete.pdf`

$\square$