Computability and Complexity

Instituto de Matemáticas

Homework

Finite automatas and regular languages

Exercise 1 : ***Reverting.***
Prove that if L is a regular language, then L^R (the language consisting in all the words in L in reverse) is regular.
You may reuse this result in some of the following exercises.

$\square$

Exercise 2 : ***Multiples of 4.***
Let $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ (thus Σ^+ is the set of nonnegatives integers written in decimal). Prove that $L = \{w \in \Sigma^* \mid w \equiv 0 \bmod 4\}$ (the language of the multiples of 4) is regular.

$\square$

Exercise 3 : ***Intersectionality.***
Prove that the regular languages are closed under intersection (*i.e.*, if L_1 and L_2 are regular languages, then $L_1 \cap L_2$ is a regular language).

$\square$

Exercise 4 : ***Comparing sums.***
Let $\Sigma_4 = \left\{ \begin{bmatrix}0\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\\1\end{bmatrix}, \begin{bmatrix}0\\0\\1\\0\end{bmatrix}, \dots, \begin{bmatrix}1\\1\\1\\1\end{bmatrix} \right\}$. Thus, any word w on Σ_4^* induces four binary numbers, called w_1, w_2, w_3, w_4 (from top to bottom).
Prove that $L = \{w \in \Sigma_4^* \mid w_1 + w_2 = w_3 + w_4\}$ is regular.

$\square$

Exercise 5 : ***One state to accept them all.***
Prove that every nondeterministic finite automata can be converted to an equivalent one that has a single accept state.

$\square$

Exercise 6 : ***Pumping.***
Use the pumping lemma to prove that $L = \{a^m b^n \mid m > n\}$ is not regular.

$\square$

Exercise 7 : ***Huey, Dewey and Louie.***
Let $\Sigma_2 = \left\{ \begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix} \right\}$. Thus, any word w on Σ_2^* induces two binary numbers, called w_1 and w_2 (from top to bottom).

1. Prove that $L = \{w \in \Sigma_2^* \mid 3w_1 = w_2\}$ (*i.e.*, where the bottom row is three times the top row) is regular.
2. Prove that $L = \{w \in \Sigma_2^* \mid w_1 = w_2^R\}$ (*i.e.*, where the bottom row is the reverse of the top row) is not regular.

$\square$