Algorithms for the Metric Dimension problem on directed graphs

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Séminaire Algo du GREYC
February 28, 2023

Where does it come from?

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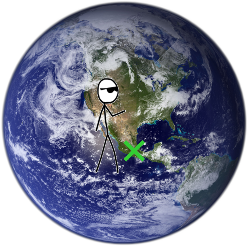

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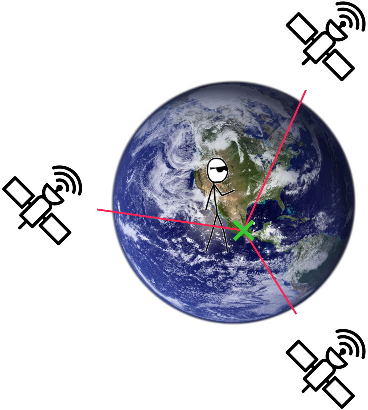

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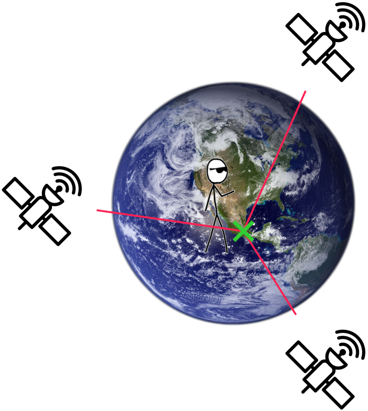

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Question

How can we transpose this approach to graphs?

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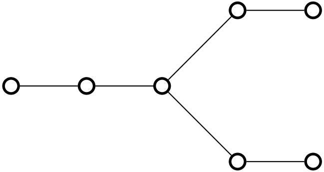

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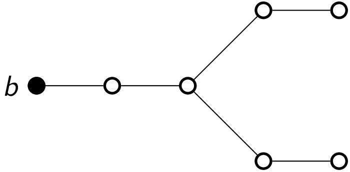

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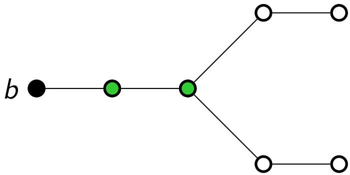

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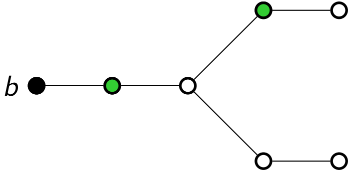

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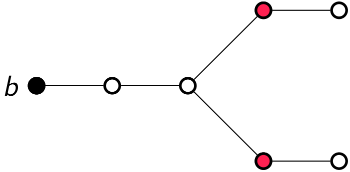

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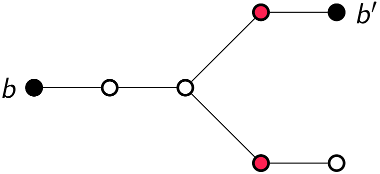

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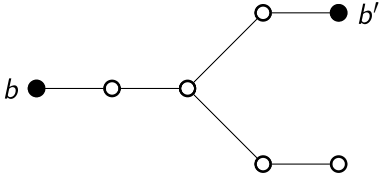

Resolving Set [Slater, 1975] [Harary & Melter, 1976]

$R \subseteq V(G)$ is a resolving set of G iff for every pair $\{u,v\}$, there is $b \in R$ that resolves u and v

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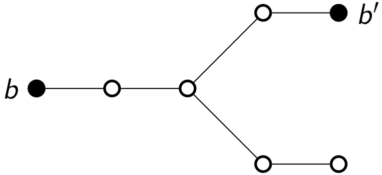

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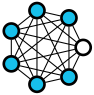

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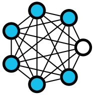

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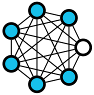

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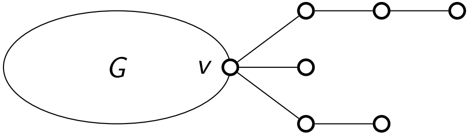

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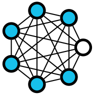

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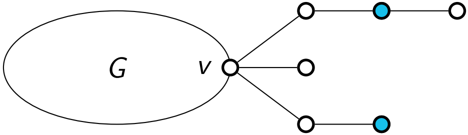

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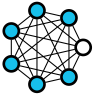

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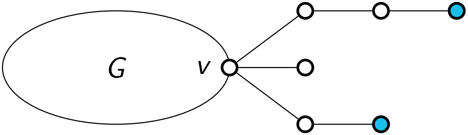

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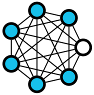
2. $\mathrm{MD}(G) = n-1 \Leftrightarrow G$ is K_n 
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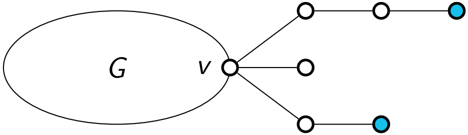

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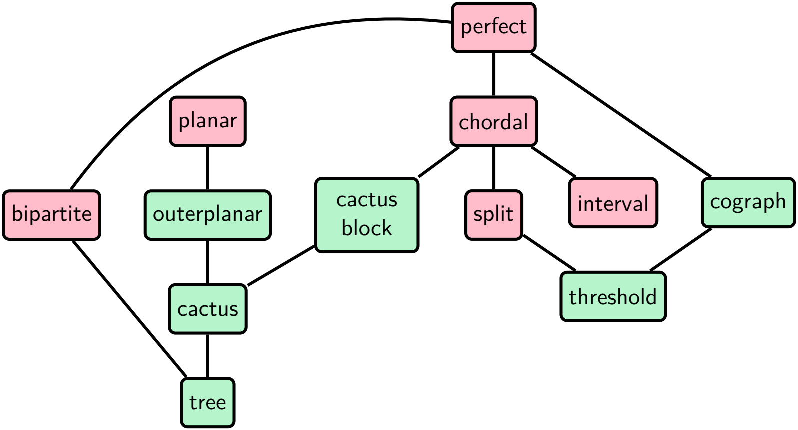
A few positive results...

- Linear-time: cographs [Epstein *et al.*, 2012], cactus block graphs [Hoffmann *et al.*, 2016]
- Polynomial-time: outerplanar graphs [Díaz *et al.*, 2012]
- FPT for bounded treelength [Belmonte *et al.*, 2015]

Inclusion diagram



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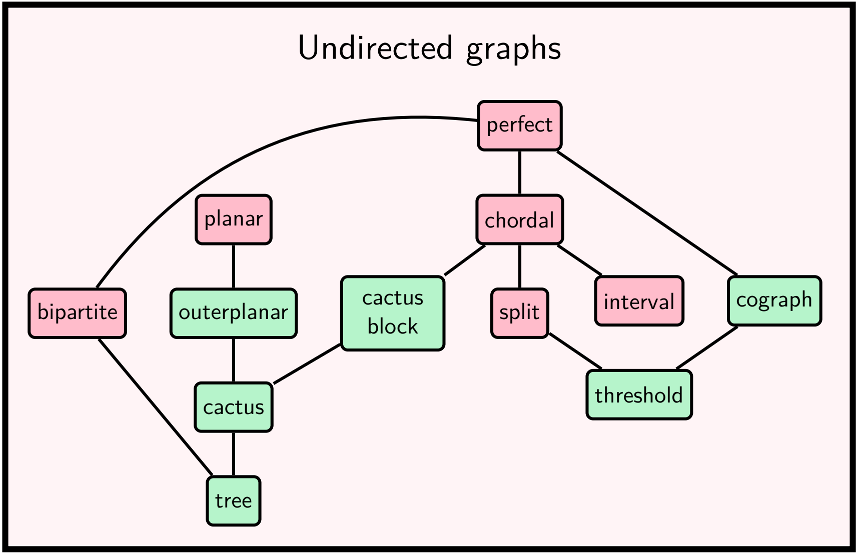

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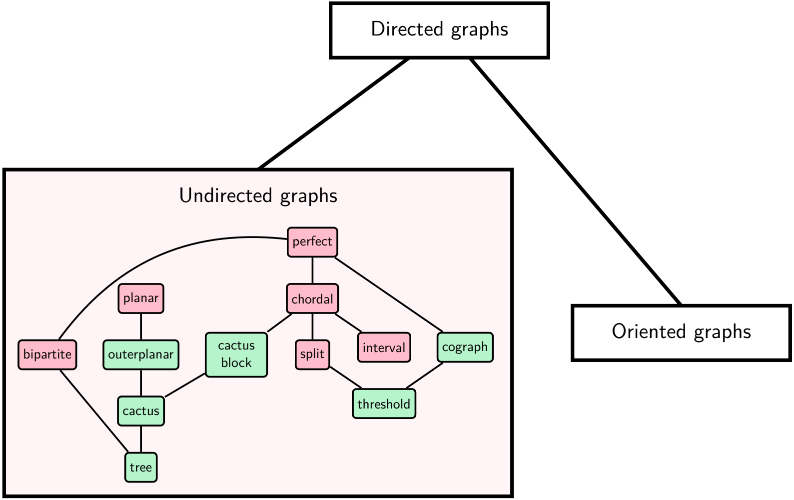
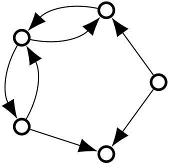


Definitions

- A directed graph may contain 2-cycles



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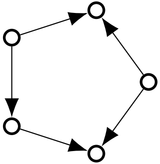

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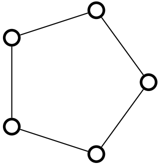

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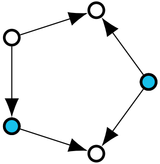

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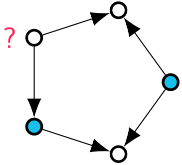

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But there will be reachability problems!

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Theorem [Araujo *et al.*, 2023+]

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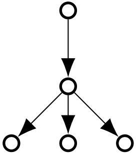

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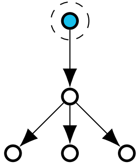

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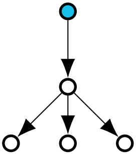

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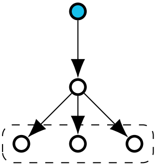

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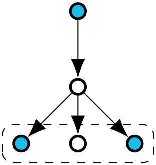

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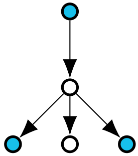

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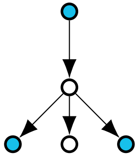

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3 The set R constructed this way is a resolving set

Our results

Theorem [D., Foucaud & Hakanen, 2023+]

Linear-time algorithms for minimum-size resolving sets of directed trees and orientations of unicyclic graphs.

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FPT algorithm parameterized by directed modular width.

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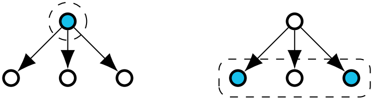

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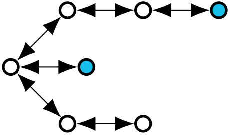
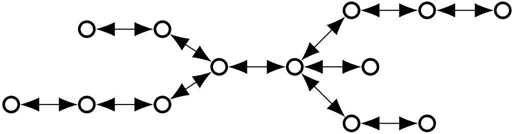
- Resolving legs of strongly connected components



Directed trees (2) Dummy vertices

Definition

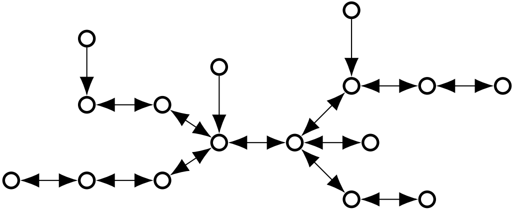
In a strongly connected component C



Directed trees (2) Dummy vertices

Definition

In a strongly connected component C, every $v \in C$ such that an arc $\overrightarrow{uv}$ with $u \notin C$ exists is a dummy vertex.



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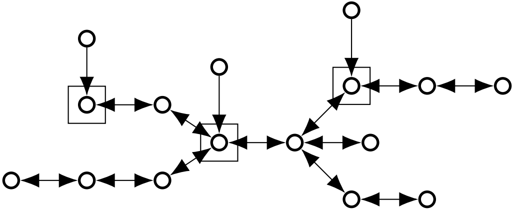

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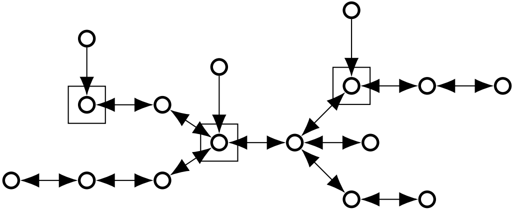

Every dummy vertex is a representative of the vertices in the resolving set behind the in-arc

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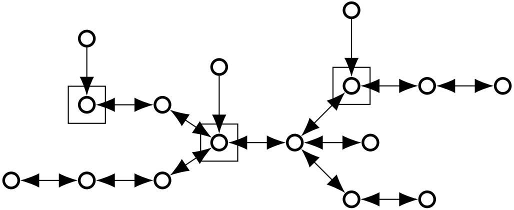


Every dummy vertex is a representative of the vertices in the resolving set behind the in-arc

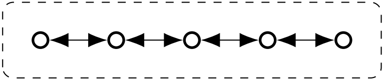
They act like degree ≥ 3 vertices for the purpose of legs

Directed trees (3) First problem: escalators

Definition

An escalator is a strongly connected component with:

- a path as an underlying graph



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An escalator is a strongly connected component with:

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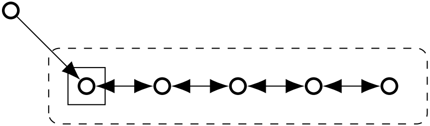
- only one in-arc from outside, at one end



Directed trees (3) First problem: escalators

Definition

An escalator is a strongly connected component with:

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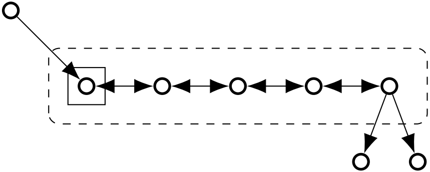
- only one in-arc from outside, at one end
- the only possible out-arcs to outside are at the other end



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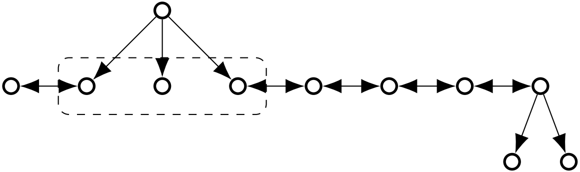

→ These are almost-in-twins

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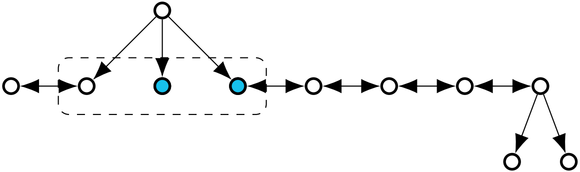


→ These are almost-in-twins

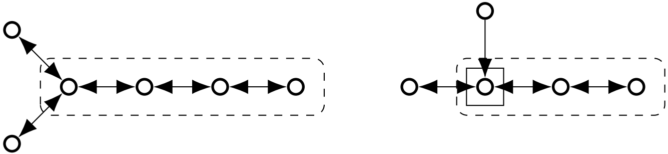
- For each set of k almost-in-twins, take $k-1$ in the resolving set

Directed trees (4) Second problem: special legs

Definition

In a strongly connected component, a special leg is a leg that:

- spans from a dummy or degree ≥ 3 (in the component) vertex



Directed trees (4) Second problem: special legs

Definition

In a strongly connected component, a special leg is a leg that:

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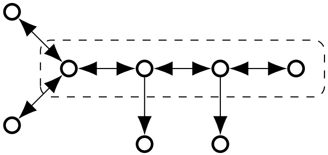
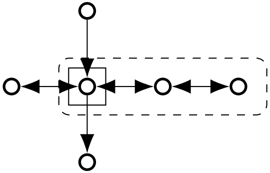
- has at least one out-arc from a vertex other than its endpoint





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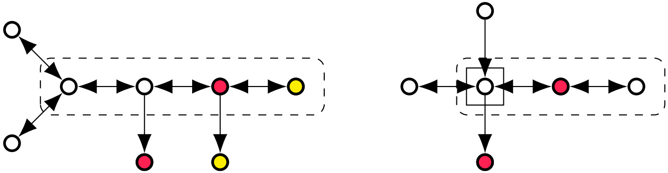

→ Conflict between pairs!

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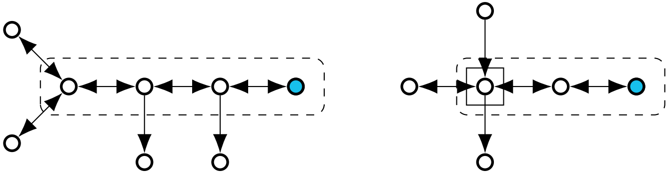
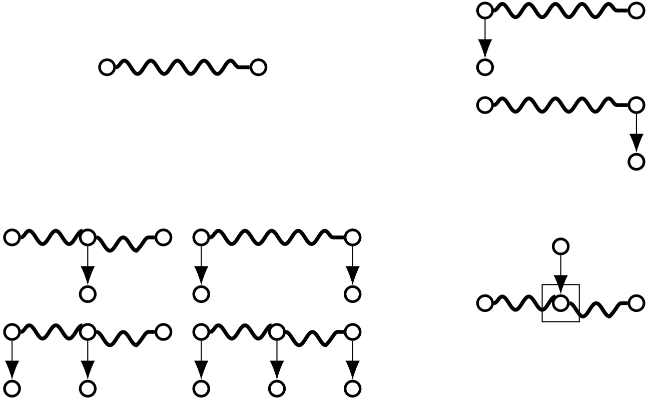


$\rightarrow$ Conflict between pairs!

- Take the endpoint of each special leg

Directed trees (5) Third problem: some paths...

The strongly connected components whose underlying graph is a path (snake = any positive length) with the following patterns:



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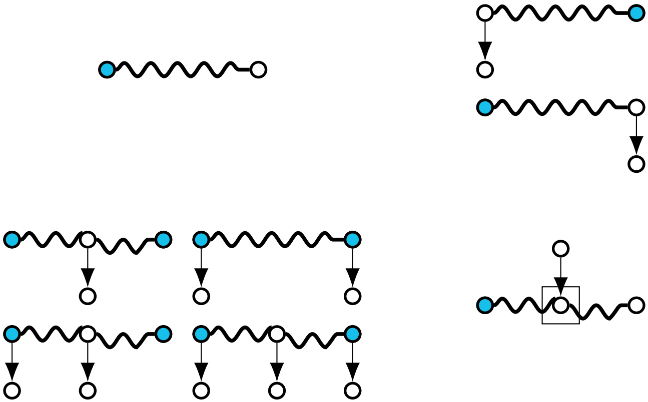

... require one or two endpoints.

Directed trees (6) The final algorithm

Theorem [D., Foucaud & Hakanen, 2023+]

There is a linear-time algorithm computing a minimum-size resolving set of a directed tree.

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   2.1 Mark the dummy vertices

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   2.4 Resolve the remaining standard legs

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1. Take every source, resolve each set of almost-in-twins
2. For each strongly connected component
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   2.3 Take the endpoint of every special leg
   2.4 Resolve the remaining standard legs

This gives a resolving set... which we prove is minimum-size!

Orientations of unicyclic graphs

Theorem [D., Foucaud & Hakanen, 2023+]

There is a linear-time algorithm computing a minimum-size resolving set of the orientation of a unicyclic graph.

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3. Resolve each set of in-twins with some priority

Orientations of unicyclic graphs

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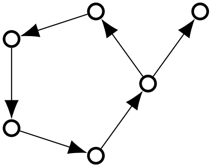
Algorithm

1. Take every source
2. Manage a few special cases (at most one more vertex)
3. Resolve each set of in-twins with some priority

No sink in the cycle



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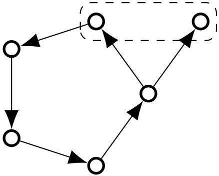
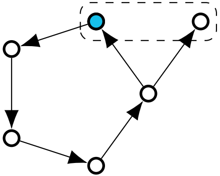


Which in-twin?

No sink in the cycle



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Priority

Give priority to in-twins in the cycle

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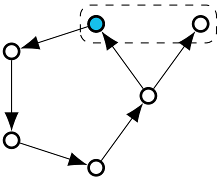

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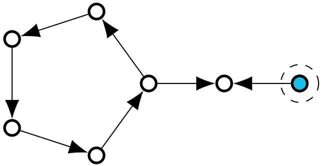

⚠ **Special case** (Reachability)

Priority

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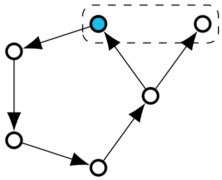

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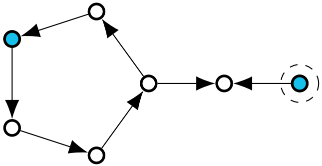

⚠**Special case** (Reachability)
$\Rightarrow$ Take one vertex from the cycle

Priority

Give priority to in-twins in the cycle

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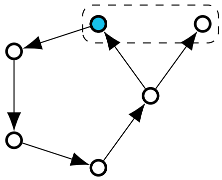

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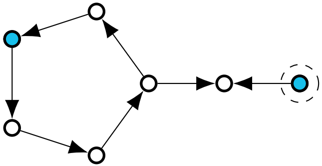

⚠ **Special case** (Reachability)
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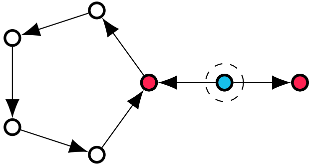

⚠ **Special case** (Unresolved pair)

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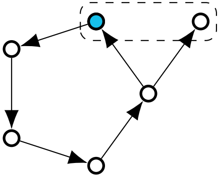

Which in-twin?

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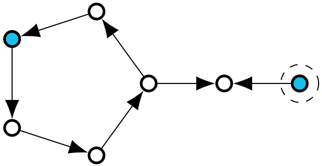

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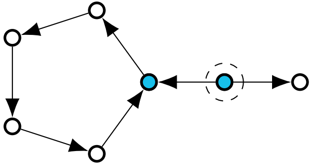
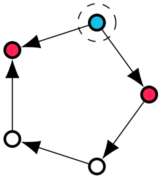


⚠**Special case** (Unresolved pair)
⇒ Take one unresolved vertex

One sink in the cycle



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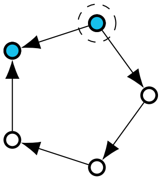
(Unresolved pair)

One sink in the cycle



⚠**Special case**
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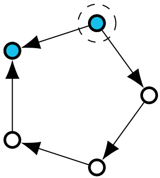

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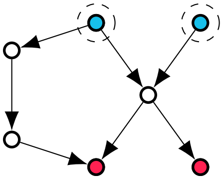

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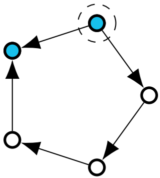

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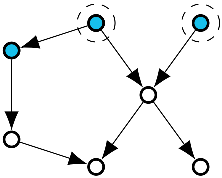

⚠**Special case**
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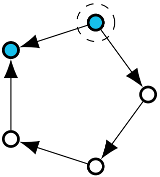
⇒ Take one vertex
along the long path

One sink in the cycle



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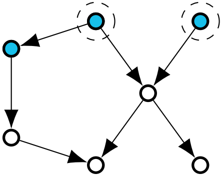

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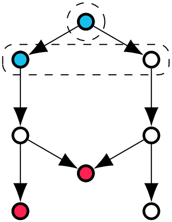

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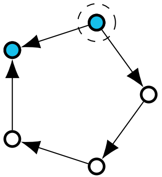

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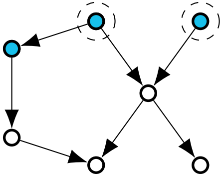

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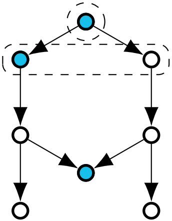
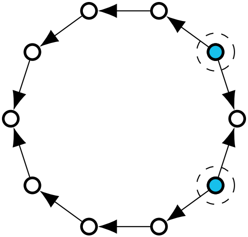


⚠**Special case**
(Unresolved pair)
⇒ Take the sink of
the cycle

Two sinks in the cycle



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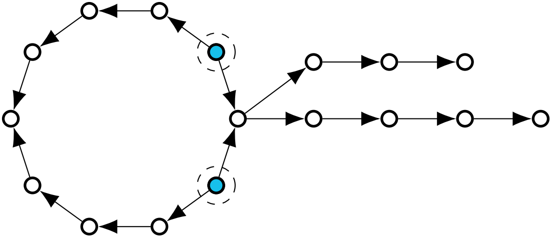

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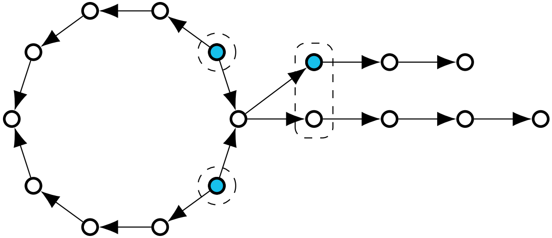

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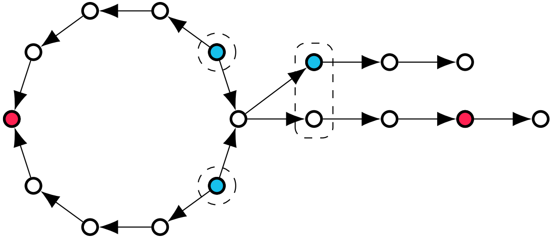

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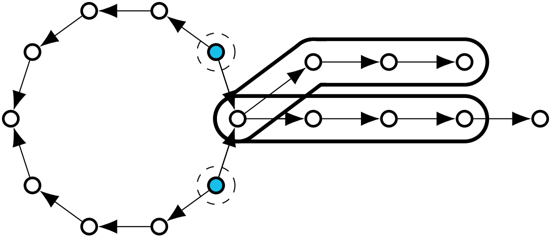
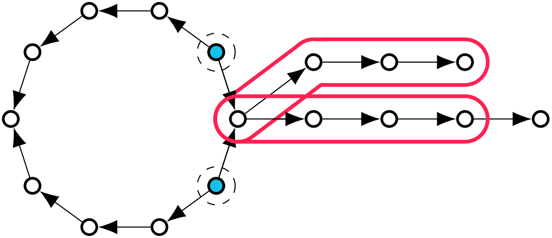
Two sinks in the cycle



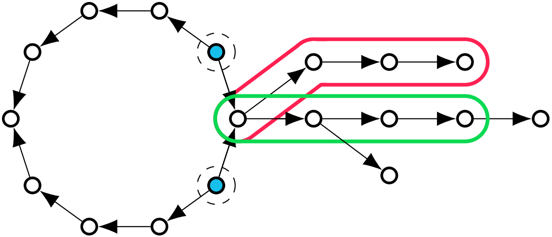
Those are **concerning paths**,

Two sinks in the cycle



Those are **concerning paths**,
which can be either unfixable

Two sinks in the cycle



Those are **concerning paths**,
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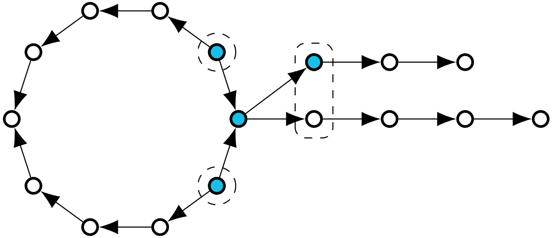
Two sinks in the cycle

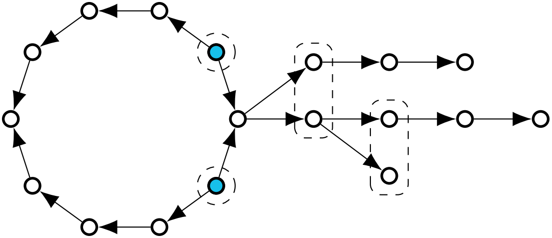


Those are **concerning paths**,
which can be either unfixable or fixable.

Special case & Priority

- If all the concerning paths are unfixable, then, take the sink

Two sinks in the cycle



Those are **concerning paths**,
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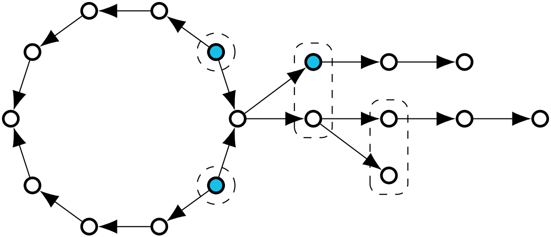

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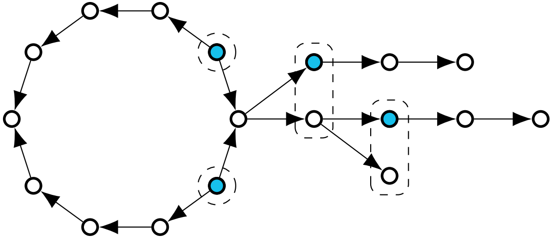

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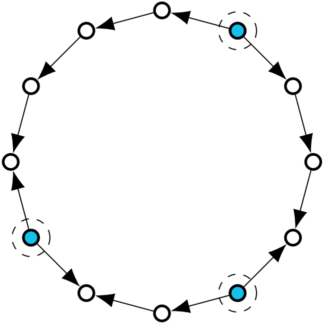

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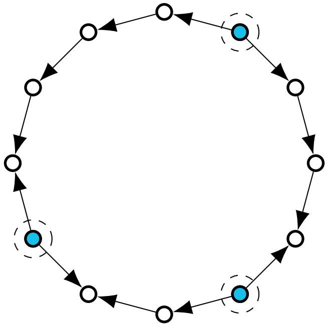

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Linear-time algorithm

1. Take every source
2. Manage the special cases
3. Resolve each set of in-twins with some priority

NP-hardness (1) The gadgets

Theorem [D., Foucaud & Hakanen, 2023+]

DIRECTED METRIC DIMENSION is NP-complete for planar triangle-free DAGs of maximum degree 6.

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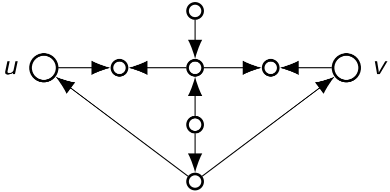

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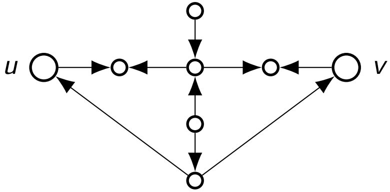

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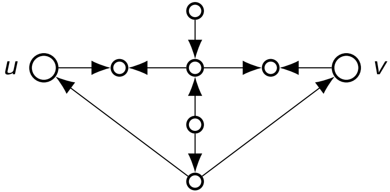

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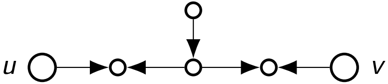

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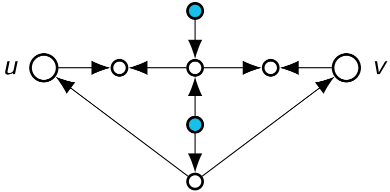

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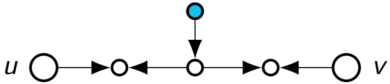

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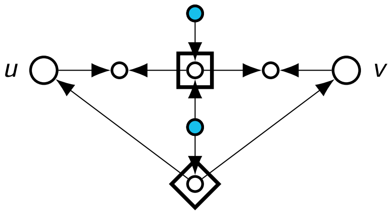

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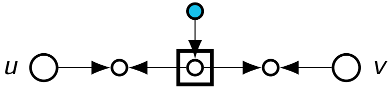

NP-hardness (2) Combining gadgets

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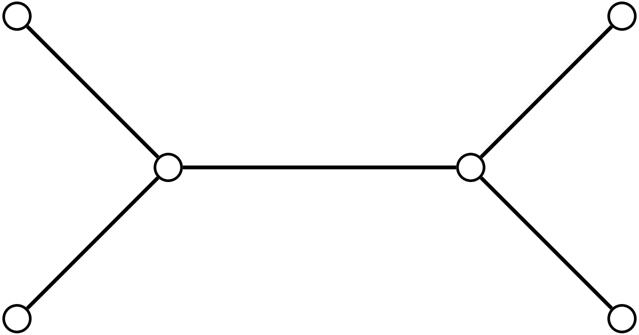

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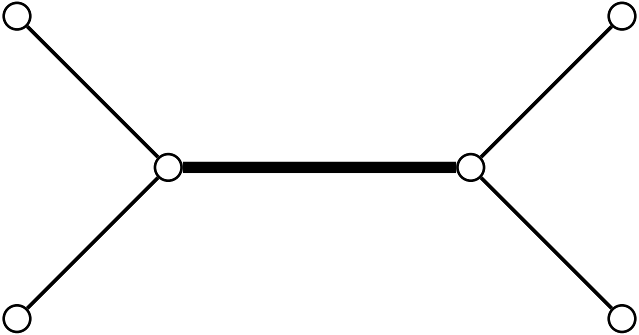

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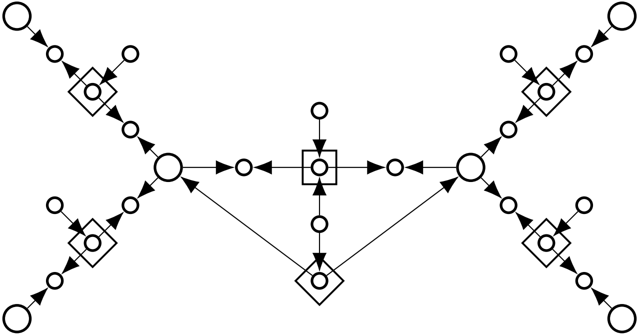

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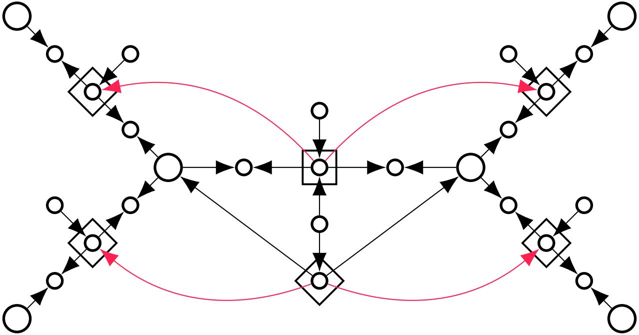

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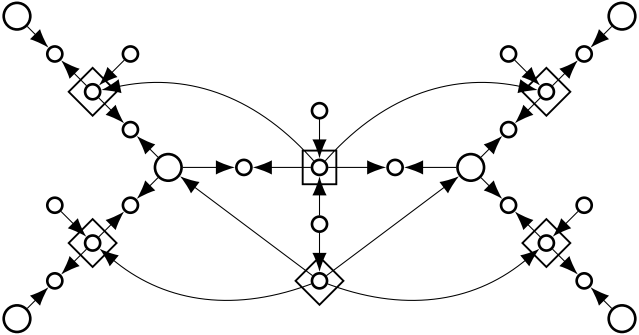

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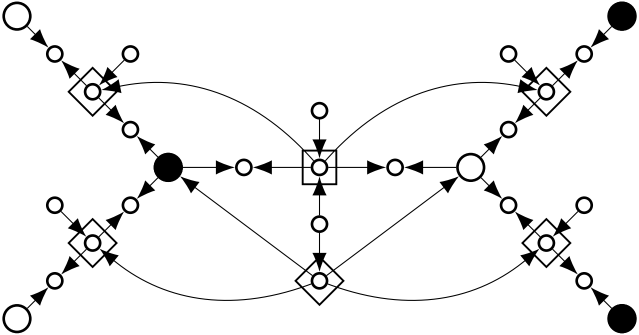

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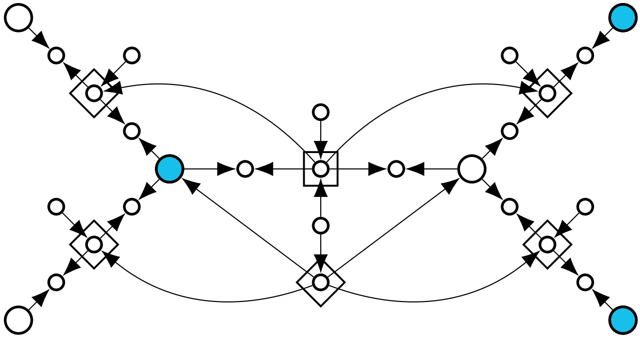

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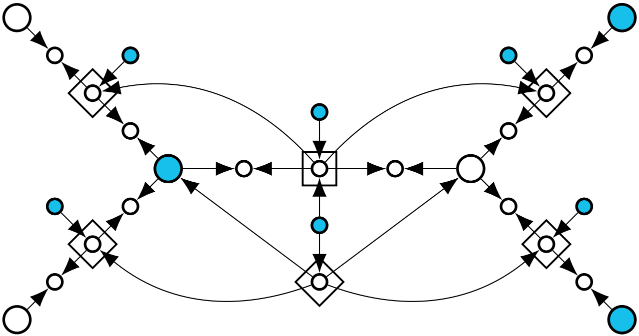

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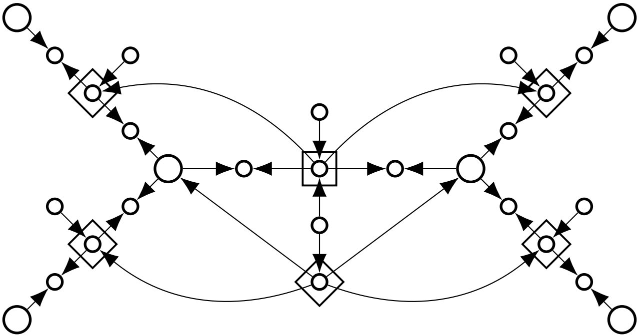

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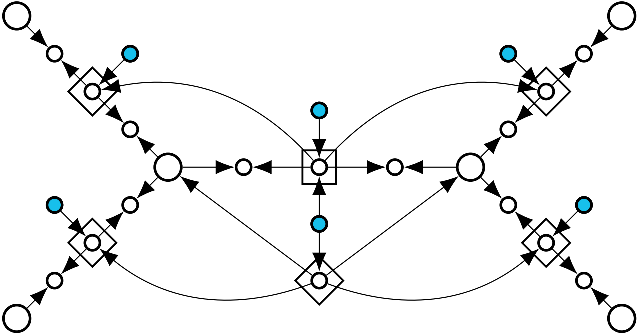

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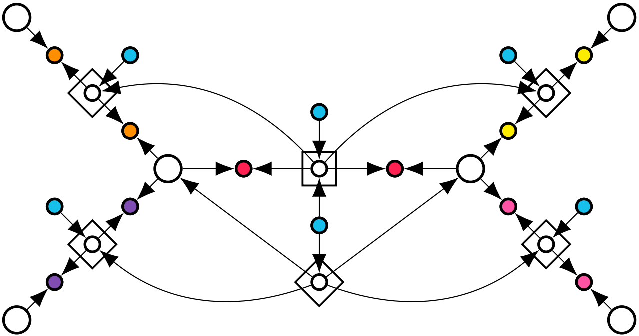

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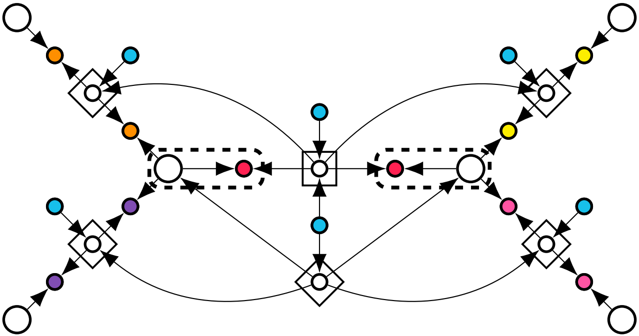

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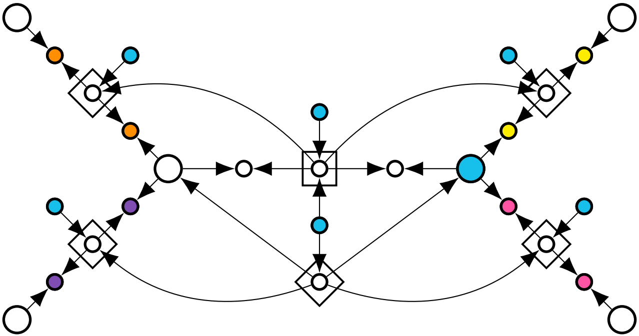

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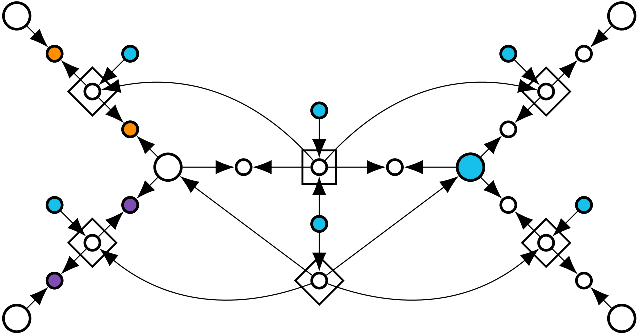

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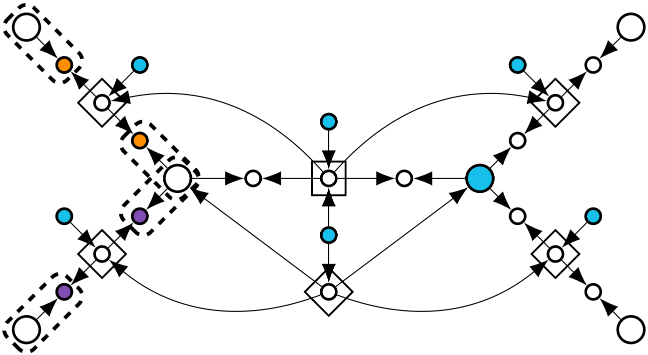

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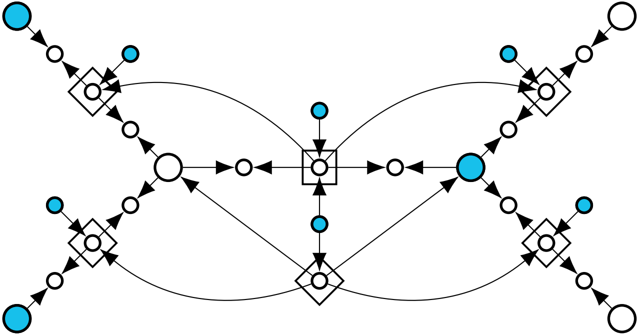

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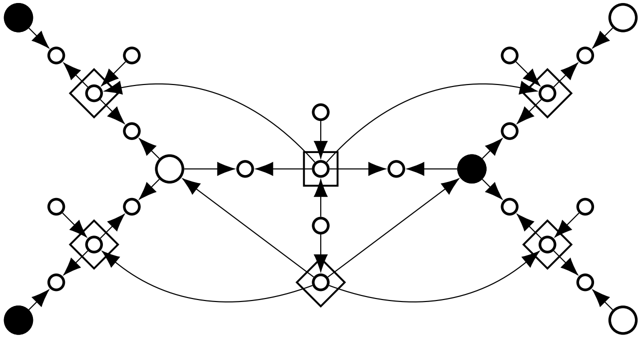

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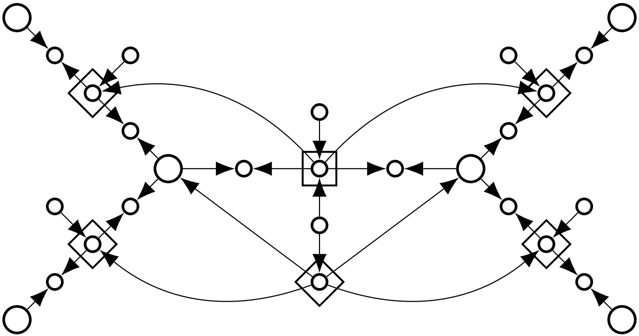

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Theorem [D., Foucaud & Hakanen, 2023+]

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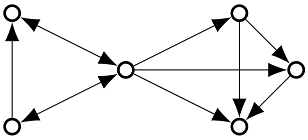
Definition [Gallai, 1967] (and many others)

A module is a set X of vertices such that every vertex **outside** of X sees vertices of X in the same way.



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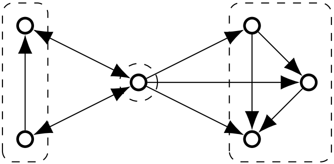

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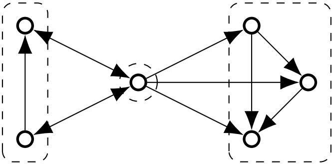

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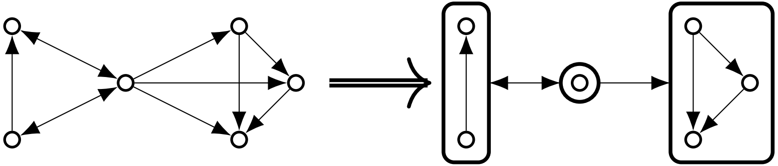

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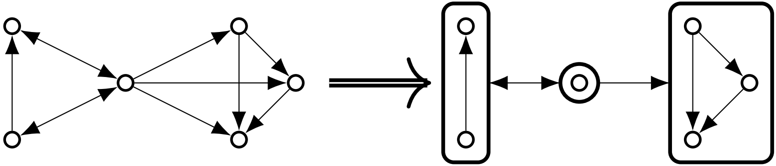

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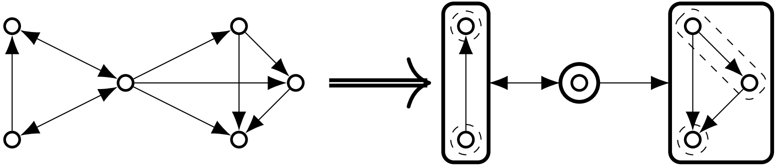

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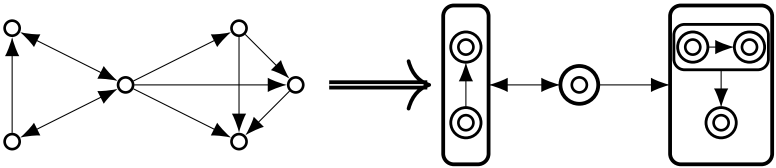

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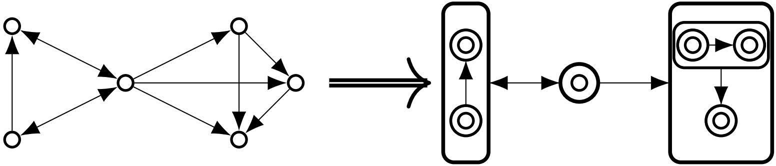

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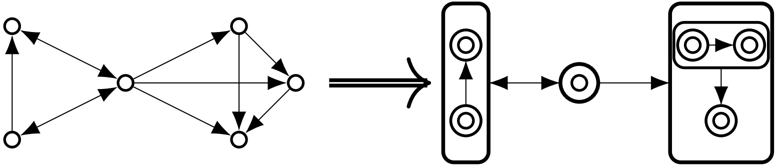

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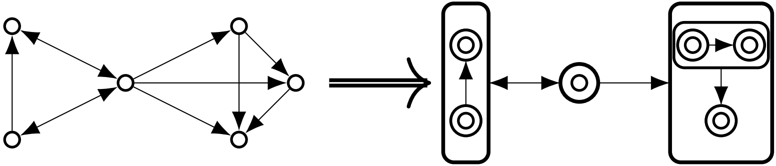

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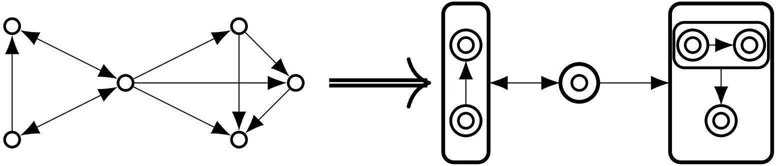
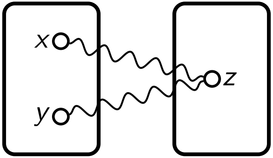


mw 3 width 3

Properties of the modular decomposition

1. Given vertices $x, y \in M_i$ and $z \in M_j$,



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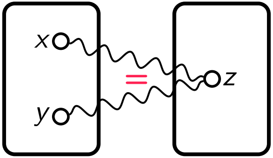

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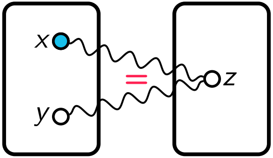

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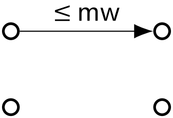

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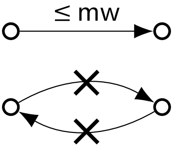

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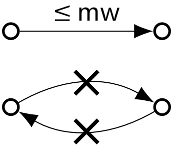

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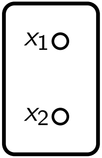

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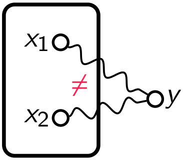

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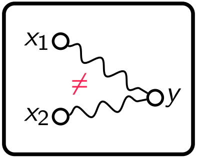

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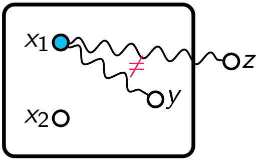

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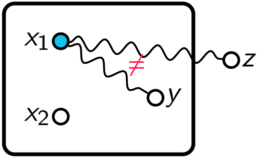

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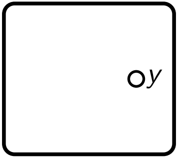

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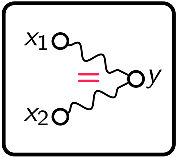

But what if, for some $y \in M_i$,
$\text{dist}(x_1,y) = \text{dist}(x_2,y)$ for every $x_1, x_2 \in M_i$?

d-constant vertices

Definition

In a module M, a vertex x is d-constant if $\mathrm{dist}(w, x) = d$ for every $w \in M_R$ (where M_R is the local solution).

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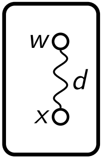

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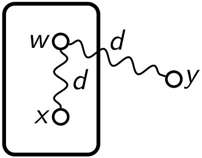

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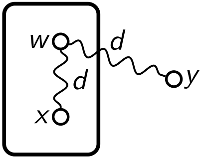

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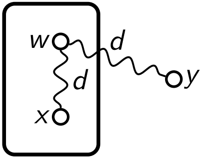

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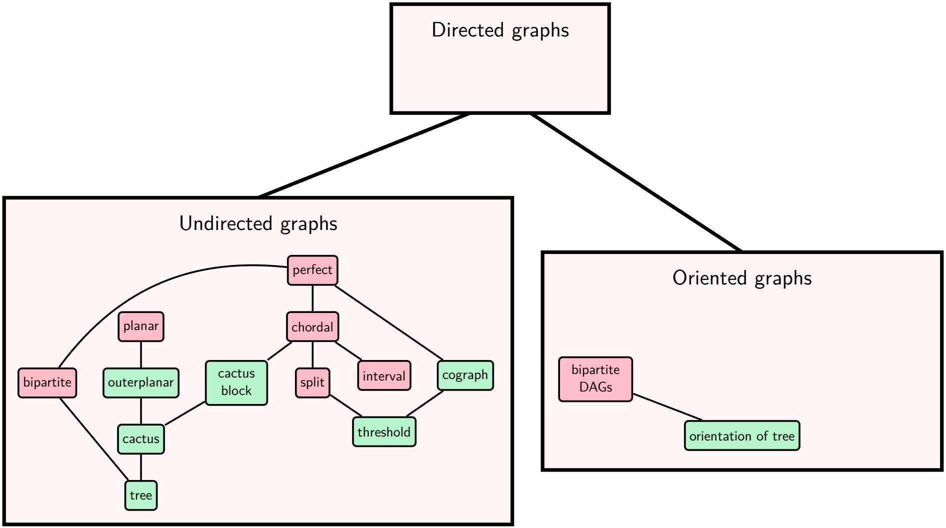
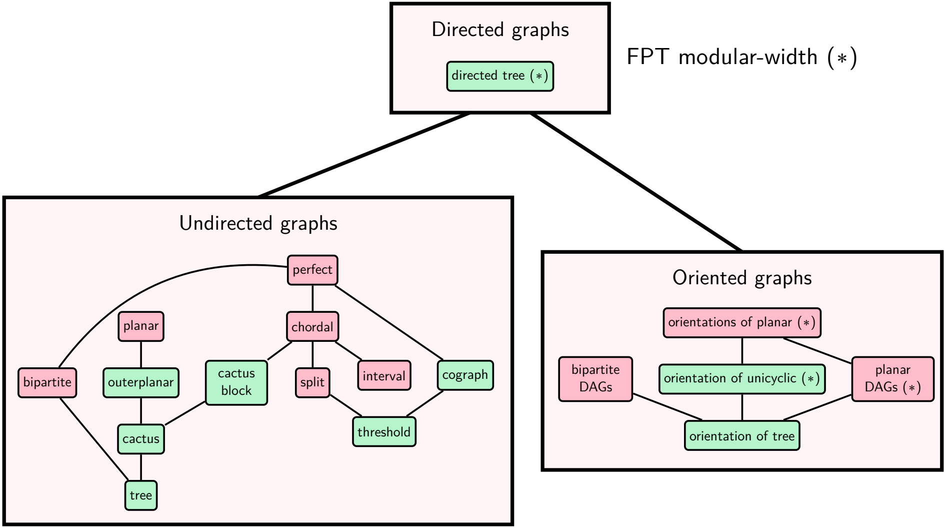
... but $d \in \{1, \ldots, \mathrm{mw}, \infty\}$ so their number is **bounded by** $\mathrm{mw}+1$ for each factor!

$\Rightarrow$ We can brute-force them when combining local solutions.

New inclusion diagram:



New inclusion diagram: $(*)$ = our results



Final words

Our contribution to Metric Dimension on directed graphs

- NP-completeness for a very restricted class
- Linear-time algorithms (directed trees, orientations of unicyclic)
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