A Canadian is traveling on an outerplanar graph...

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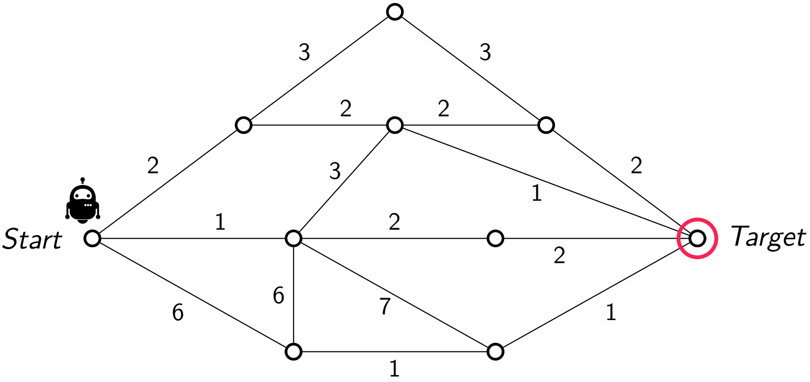
The Canadian Traveler



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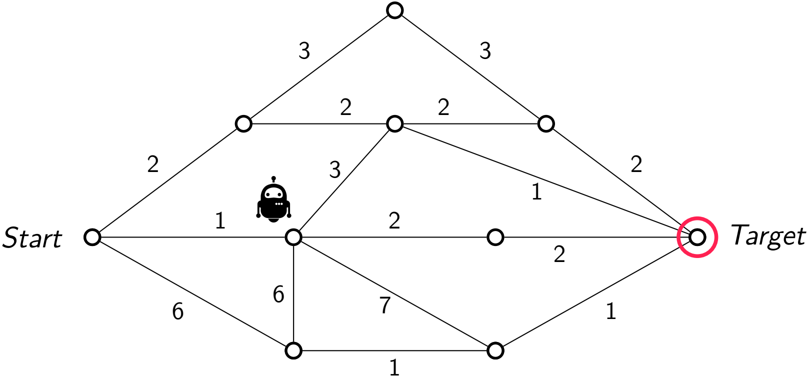

Traveled distance $= 1$

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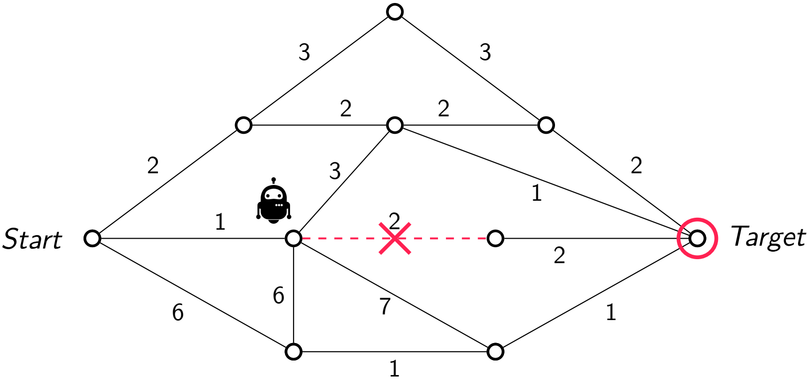

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- Some edges are **blocked**, discovered when visiting an endpoint

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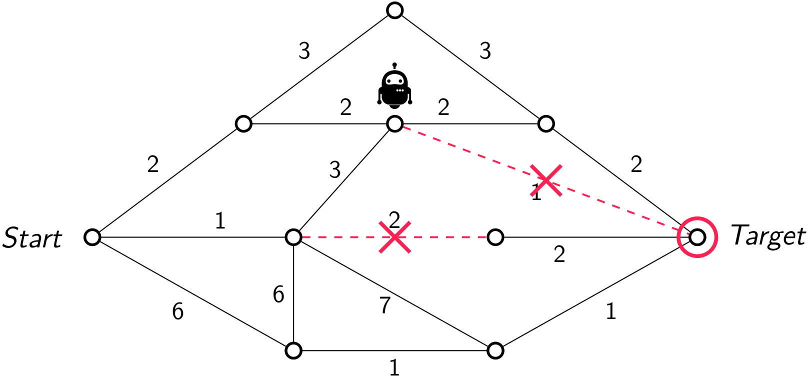

Traveled distance $= 4$

- Some edges are **blocked**, discovered when visiting an endpoint

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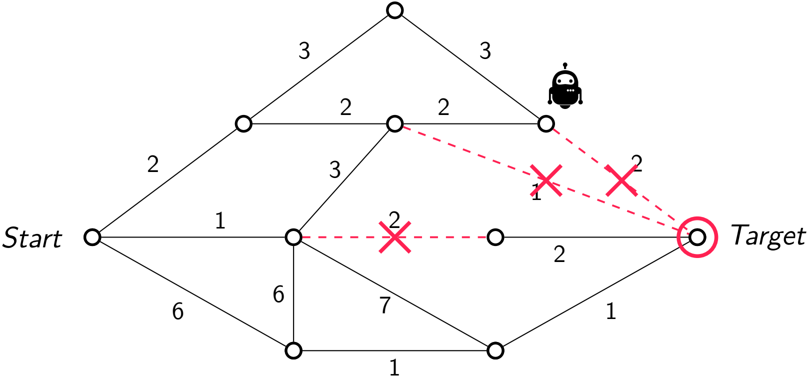

Traveled distance $= 6$

- Some edges are **blocked**, discovered when visiting an endpoint

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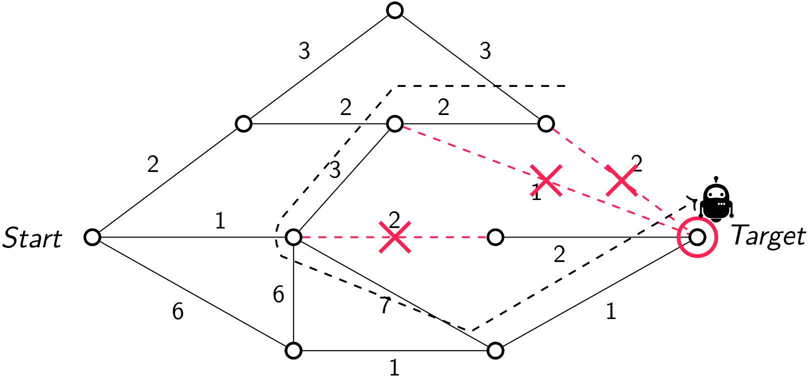

Traveled distance $= 19$

- Some edges are **blocked**, discovered when visiting an endpoint
- We can always reach the target

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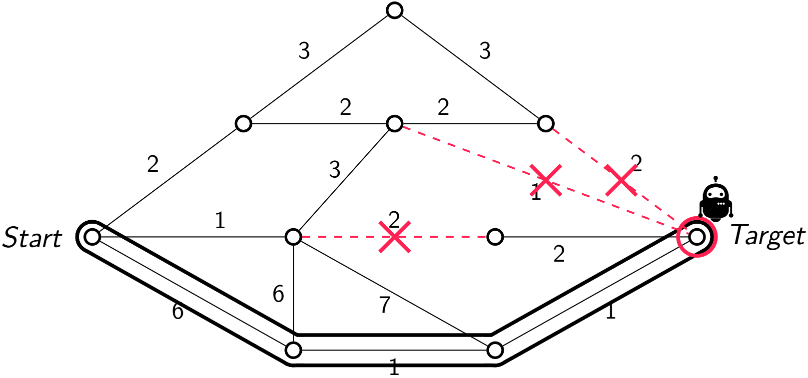

Traveled distance $= 19$
Optimal distance $= 8$

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Formal definition and bad news

k-CTP [Papadimitriou & Yannakakis, 1991]

Input: A weighted graph, two vertices s and t.
Hidden input: At most k blocked edges.
Objective: Go from s to t with minimum traveled distance.

Evaluating a strategy

Minimizing the **competitive ratio** $\frac{\text{traveled distance}}{\text{optimal distance}}$

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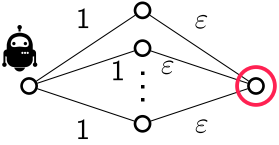

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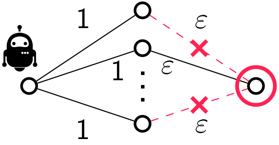

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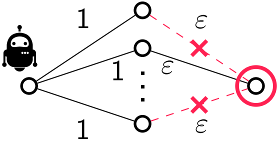

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The construction can even be made unit-weighted...

A bit of history

- k-CTP is PSPACE-complete [Papadimitriou & Yannakakis and Bar-Noy & Schieber, 1991]
- Many variants (probabilistic, multiple travelers, sensing remote edges, temporal graphs...) with applications to robot routing
- The GREEDY strategy (follow a shortest path from s to t, when blocked at x, compute a shortest path from x to t) can be arbitrarily bad
- Two deterministic strategies reach competitive ratio $2k+1$ in general graphs:

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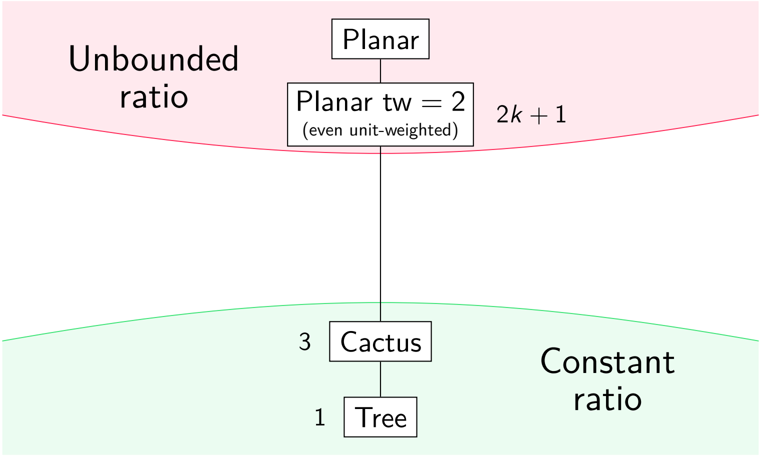
  - COMPARISON [Xu, Hu, Su, Zhu & Zhu, 2009]: trade-off between GREEDY and REPOSITION
- Non-deterministic strategies can be quite good: competitive ratio $(1 + \frac{\sqrt{2}}{2})k + O(1)$ [Demaine *et al.*, 2014], but no better than $k + 1$ [Westphal, 2008]

What now?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Unbounded ratio
Planar tw = 2
(even unit-weighted)
2k + 1
3
Cactus
Constant ratio
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Tree

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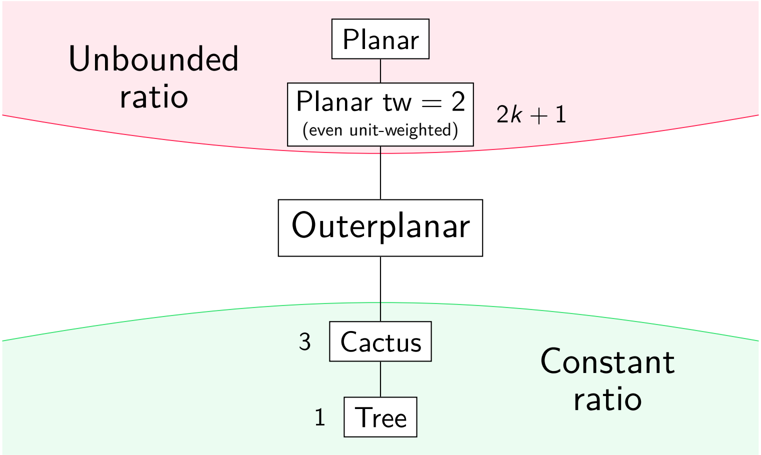
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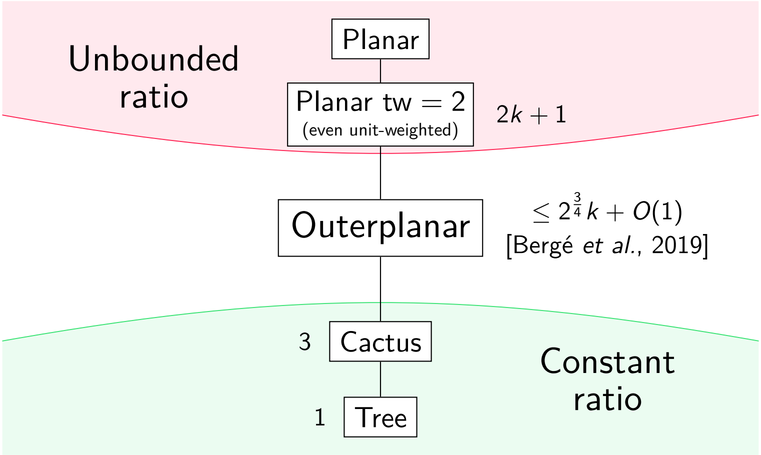

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$\leq 2\frac{3}{4}k + O(1)$
[Bergé *et al.*, 2019]
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Our results

Theorem [BBCDGLLP, 2024-2026]

There is a strategy with competitive ratio 9 for unit-weighted outerplanar graphs.

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There is a family of weighted outerplanar graphs on which no strategy can have competitive ratio less than $g(k) \in \Omega(\frac{\ln k}{\ln \ln k})$.

Shells and cows

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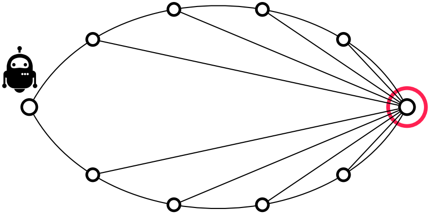

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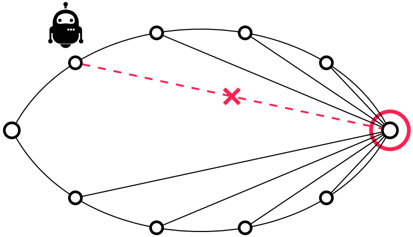

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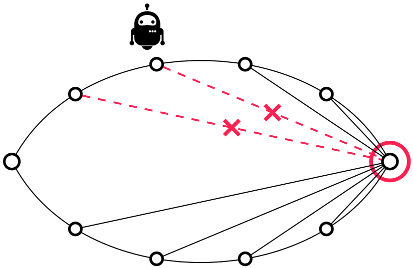

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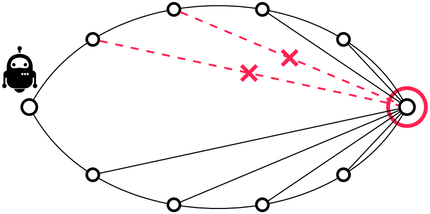

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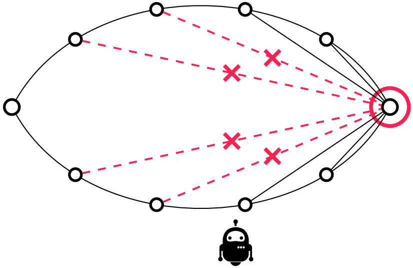

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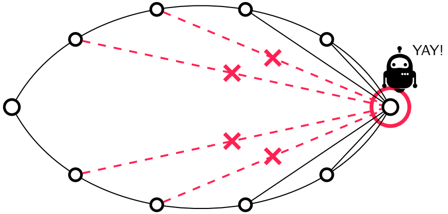

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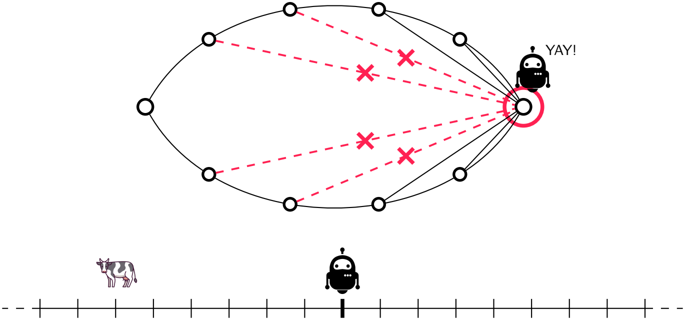

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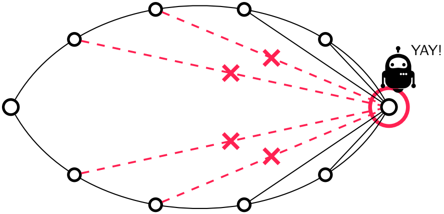

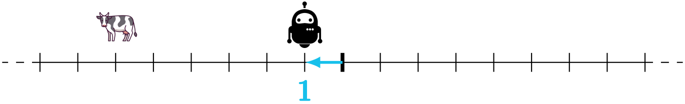

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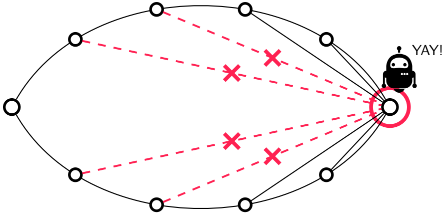

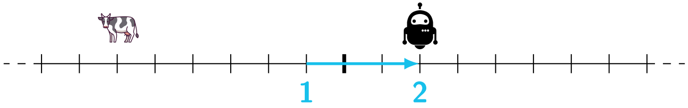

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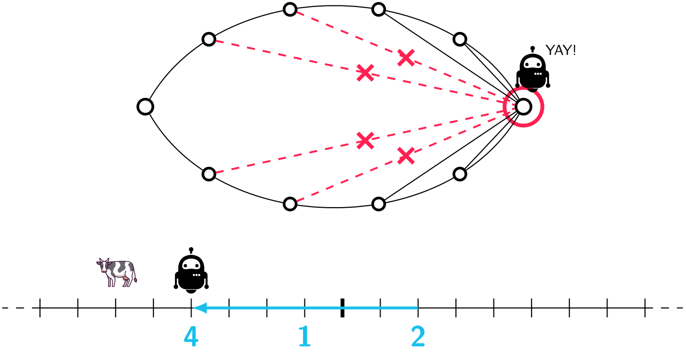

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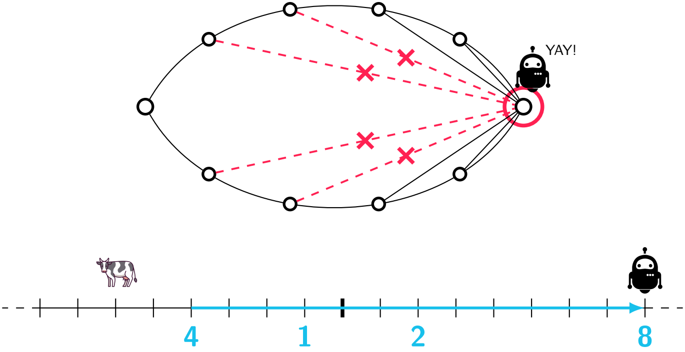

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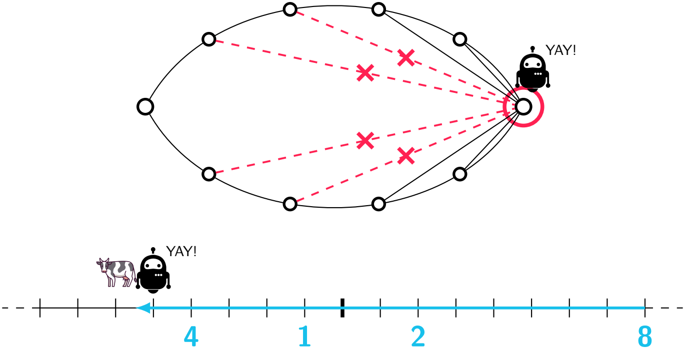

$\rightarrow$ No strategy can be better than this, which has ratio 9

Competitive ratio 9 on unweighted outerplanar graphs

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Proof Sketch

1. Manage articulation points to simplify and decompose the graph
2. Use exponential balancing while managing chords, using induction to iterate

Main idea: going from one side to the other without going back to the start can be useful!

Managing articulation points

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Lemma

Let $\mathcal{F}$ be a monotone family. If A achieves competitive ratio C on graphs of $\mathcal{F}$ with no articulation point, then, the strategy:

1. Remove all useless components
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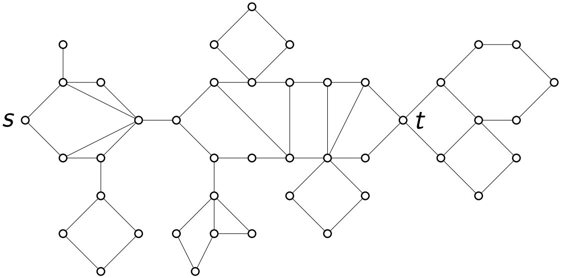

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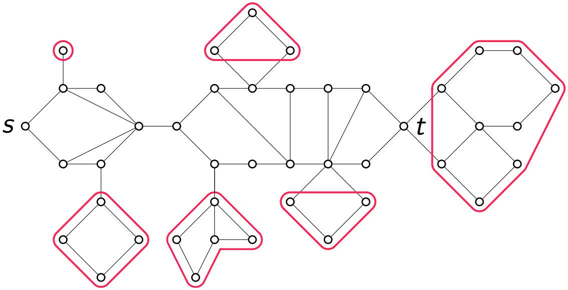

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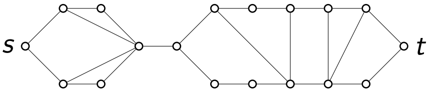

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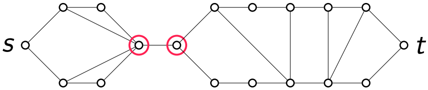

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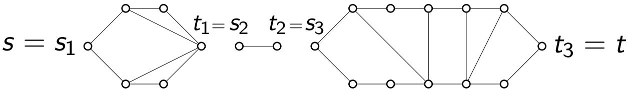
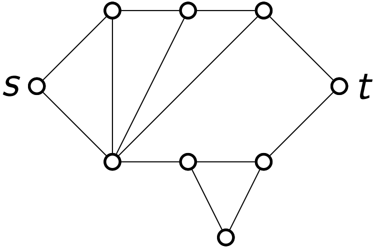


Sides and chords



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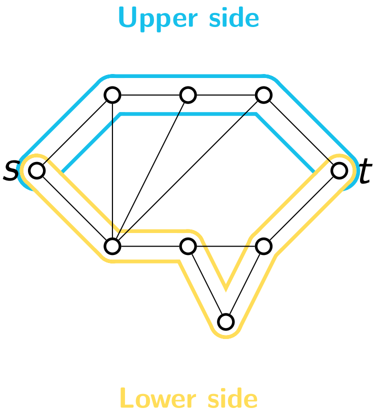

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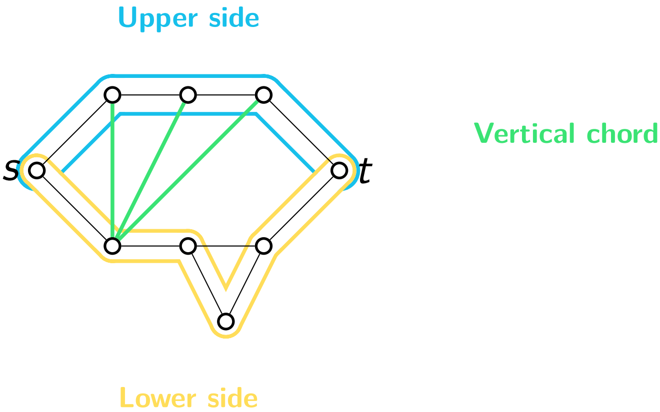

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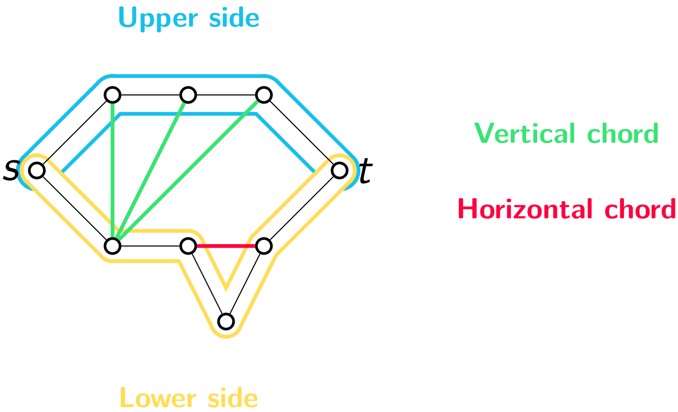

→ When following a path, we always take open horizontal chords and can ignore what they allow to skip

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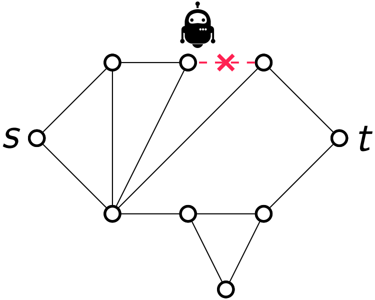

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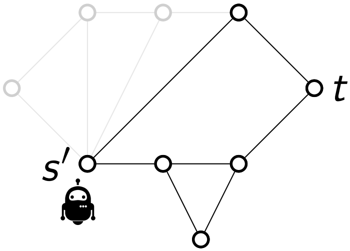
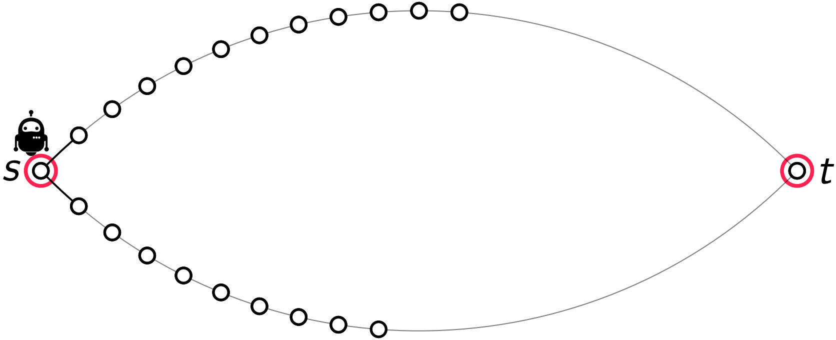
→ If one side is blocked

Sides and chords



$\rightarrow$ When following a path, we always take open horizontal chords and can ignore what they allow to skip
$\rightarrow$ If one side is blocked, then, we switch to the other and can reapply the simplification

Exponential balancing and vertical chords



Exponential balancing and vertical chords

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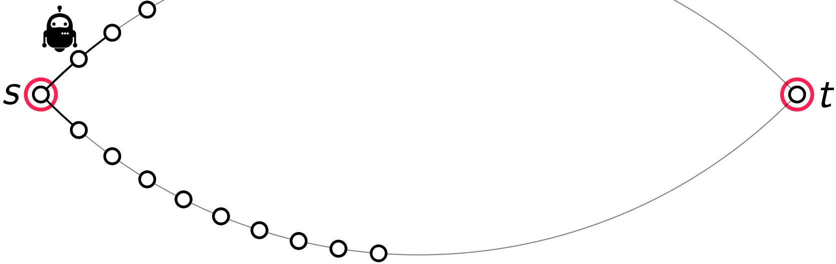

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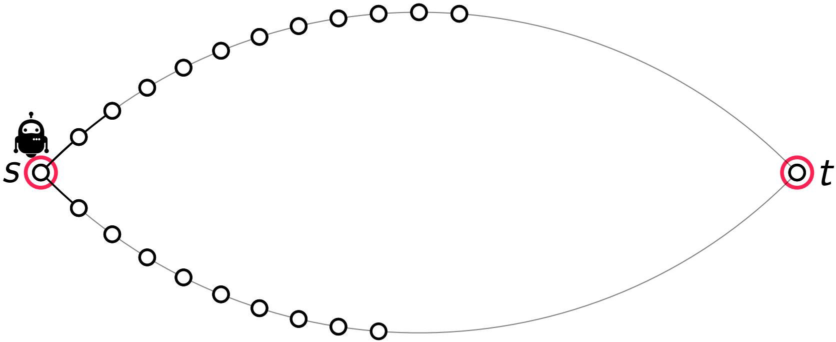

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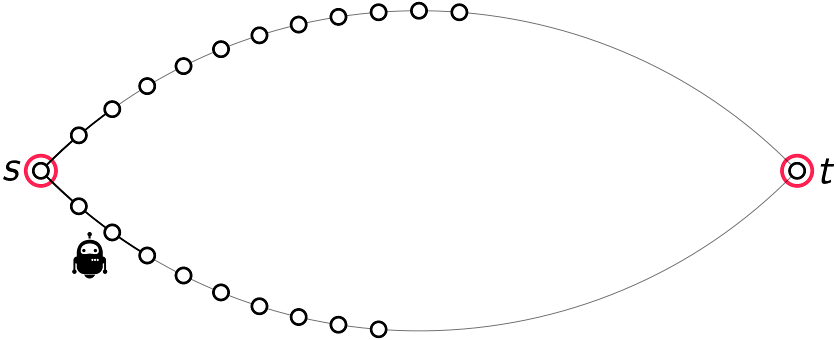
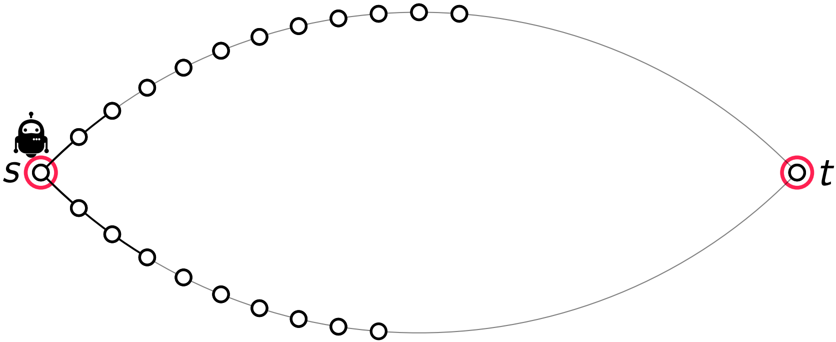


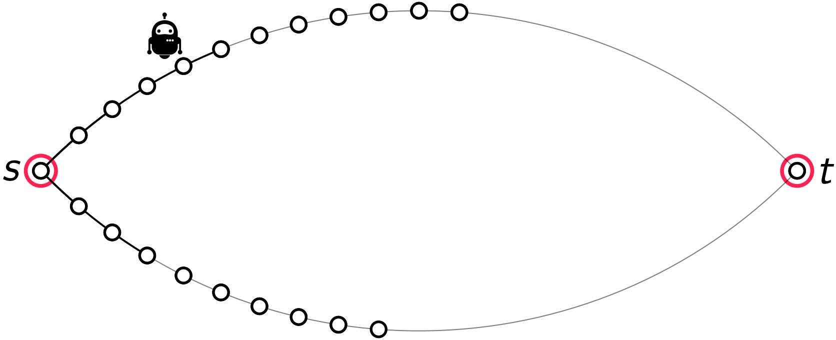
Exponential balancing and vertical chords

Budget $= 4$



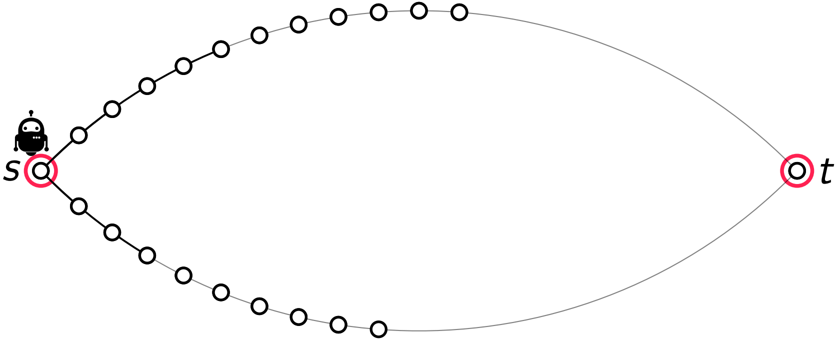
Exponential balancing and vertical chords

Budget $= 4$



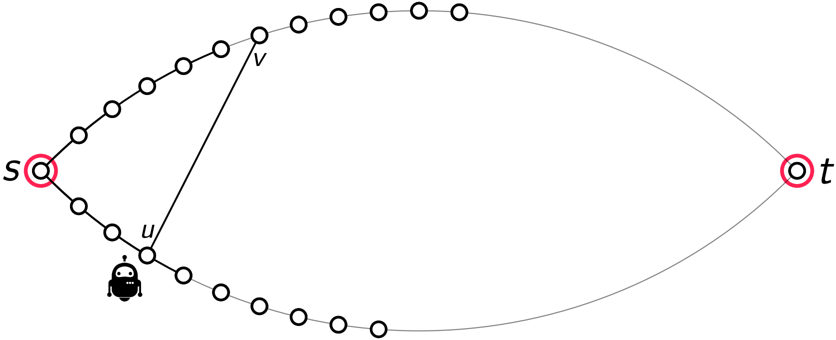
Exponential balancing and vertical chords

Budget $= 8$



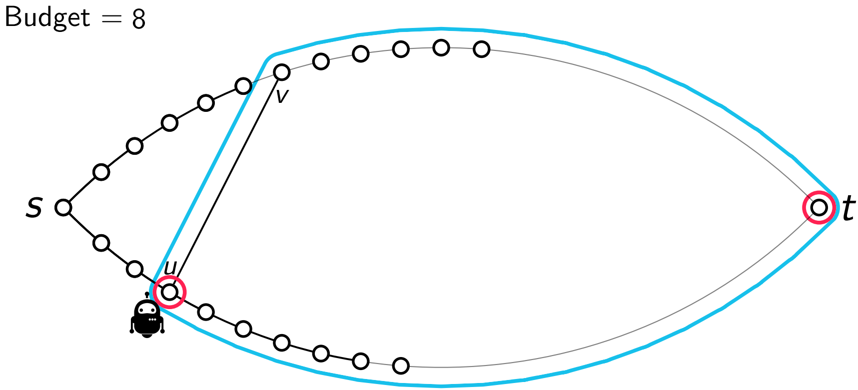
Exponential balancing and vertical chords

Budget $= 8$



Case 1: When catching up, a vertical chord allows us to go further on the other side than previous budget.

Exponential balancing and vertical chords



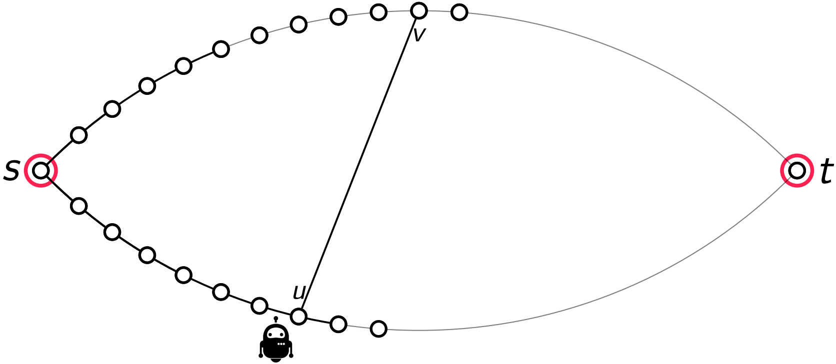
Case 1: When catching up, a vertical chord allows us to go further on the other side than previous budget.

$\Rightarrow$ Shortest *sv*-path goes through *u*

$\Rightarrow$ **Iterate** from *u* to *t*

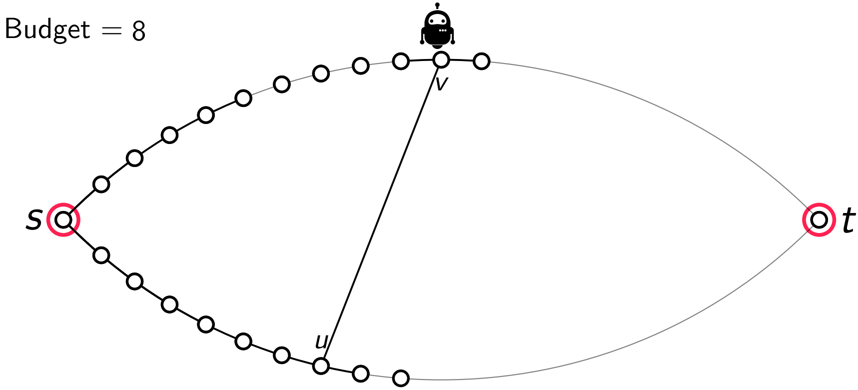
Exponential balancing and vertical chords

Budget $= 8$



Case 2: When exploring further than previous budget, a vertical chord links us to an unexplored vertex on the other side.

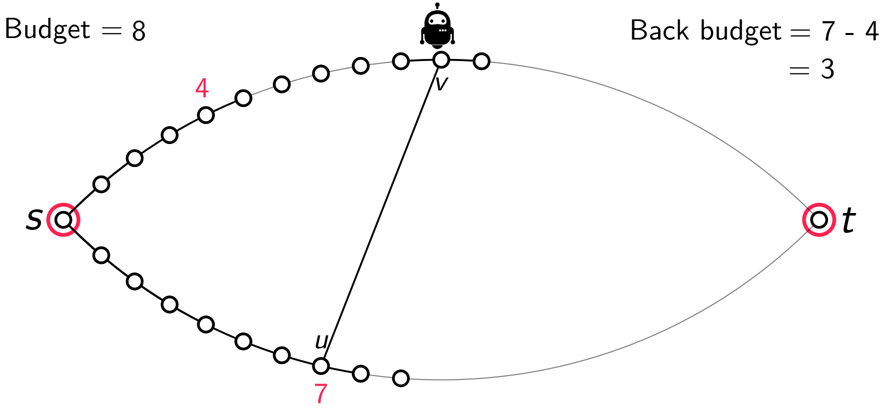
Exponential balancing and vertical chords



Case 2: When exploring further than previous budget, a vertical chord links us to an unexplored vertex on the other side.

$\Rightarrow$ Go to the other side and explore **back**

Exponential balancing and vertical chords



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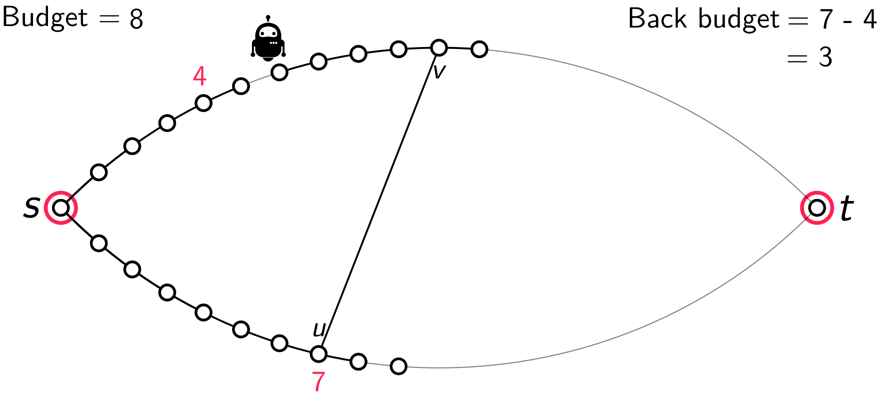
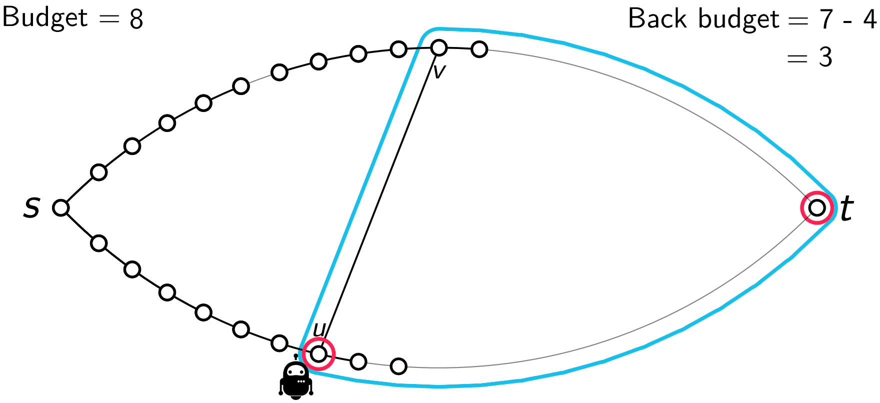


Case 2.1: We do not reach the last explored vertex on the other side.

Exponential balancing and vertical chords



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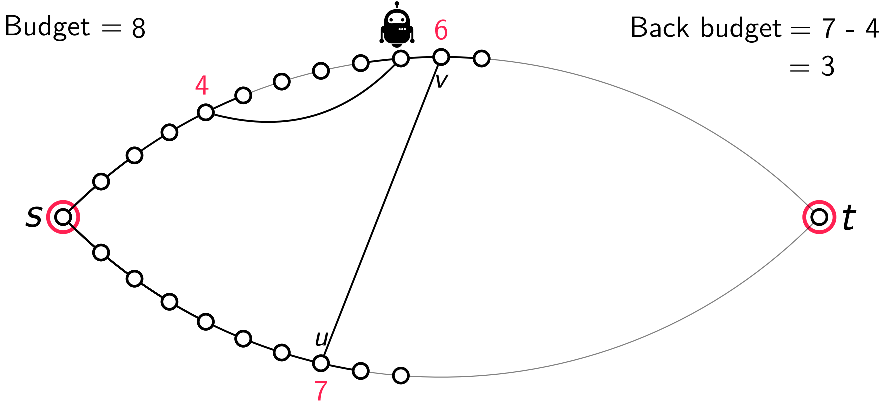
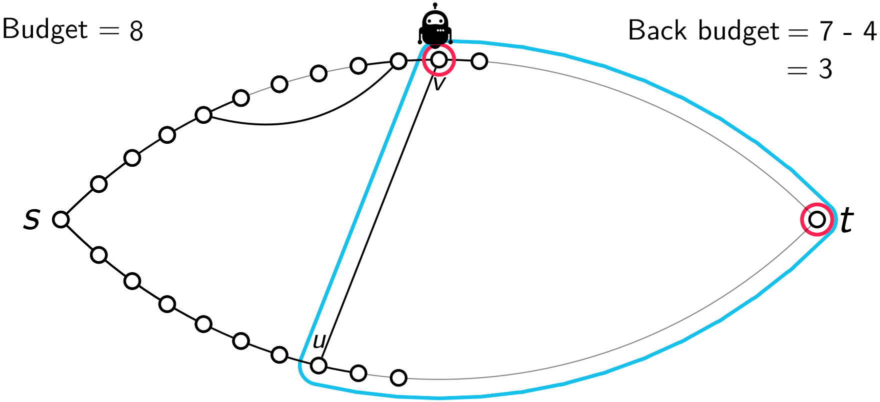


Case 2.2: We reach the last explored vertex on the other side.

Exponential balancing and vertical chords



Case 2.2: We reach the last explored vertex on the other side.

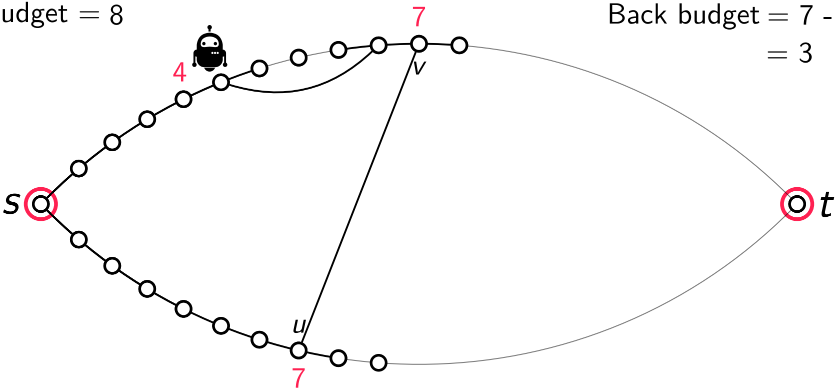
⇒ Shortest *su*-path goes through *v*

⇒ **Iterate** from *v* to *t*

Exponential balancing and vertical chords

Budget = 8

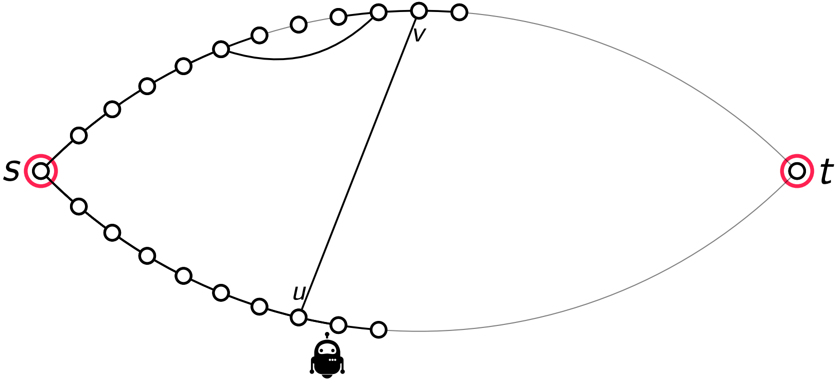
Back budget = 7 - 4
= 3



Case 2.3: u and v are at the same distance from s on their sides.

Exponential balancing and vertical chords

Budget $= 8$

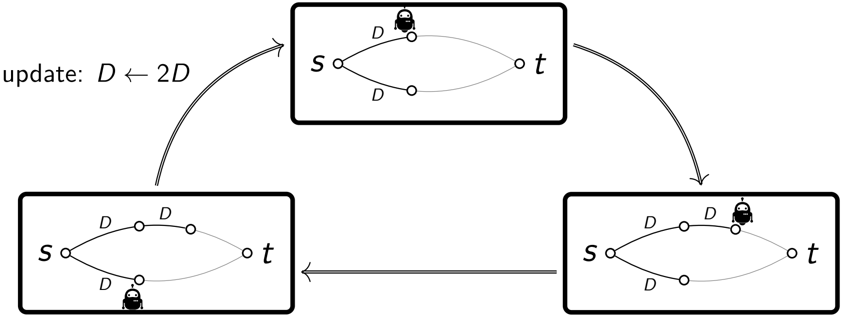


Case 2.3: u and v are at the same distance from s on their sides.

$\Rightarrow$ uv is just a shortcut between sides

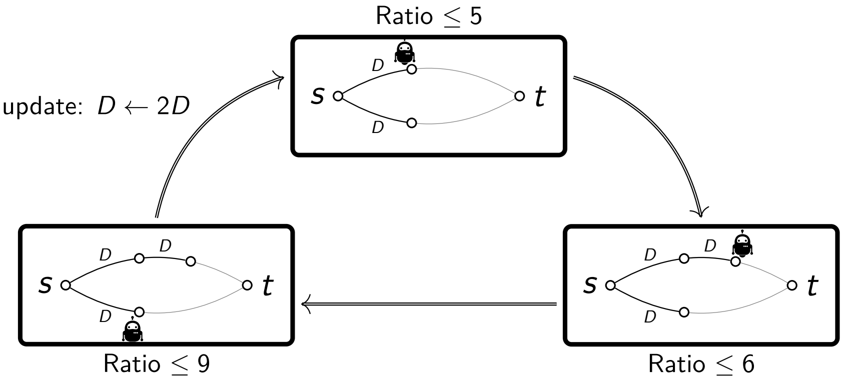
$\Rightarrow$ **Continue the balancing** from s to t

Proof of the ratio



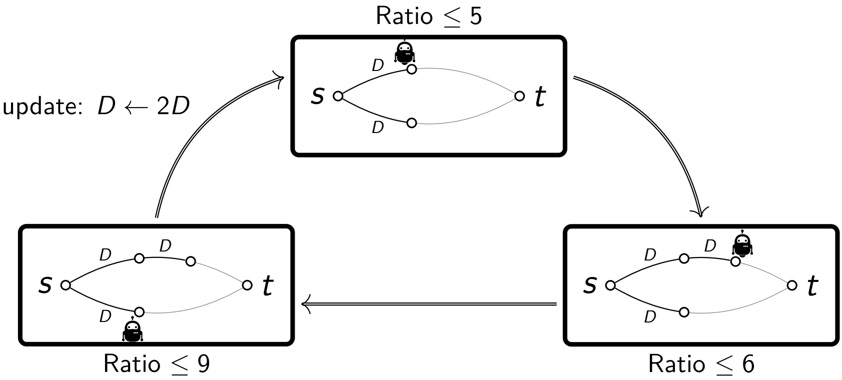
- Core **exponential balancing** loop:

Proof of the ratio



- Core **exponential balancing** loop: ratio ≤ 9

Proof of the ratio



- Core **exponential balancing** loop: ratio ≤ 9
- Going back within leftover budget **ensures** that the ratio remains ≤ 9

Arbitrarily weighted outerplanar graphs

Theorem [BBCDGLLP, 2024-2026]

There is a family of weighted outerplanar graphs on which no strategy can have competitive ratio less than $g(k) = e^{W(\frac{\ln k}{2})} - 1$.

Asymptotically, when k is large enough, $\frac{\ln k}{\ln \ln k} \leq g(k) \leq \ln k$.

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Proof

We construct H_i by induction such that, for any $\varepsilon > 0$, there is a blocking strategy using less than $((i+1)!)^2$ blocked edges such that in every strategy, the distance traversed is at least $S_i = (i+1)!$, the competitive ratio is at least $r_i^\varepsilon = i + 1 - \varepsilon$.

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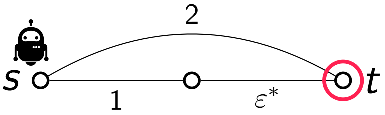
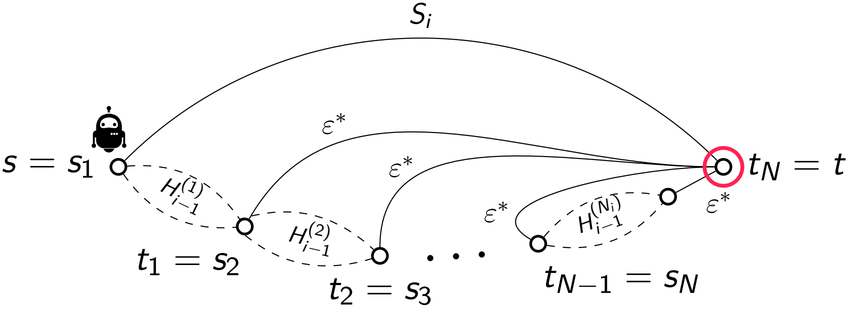
H_1:



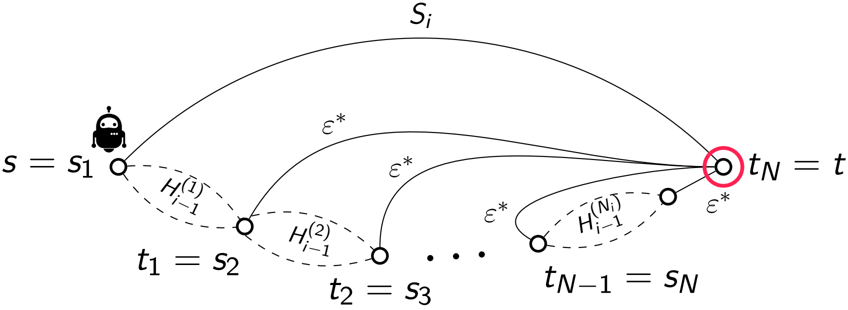
Competitive ratio $\frac{2}{1+\varepsilon^*}$ by going above (no edge blocked), and 2 by going below (small edge blocked).

Building H_i from H_{i-1}



with $N_i = i(i+1)$.

Building H_i from H_{i-1}



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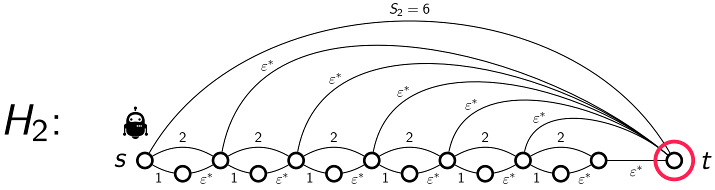
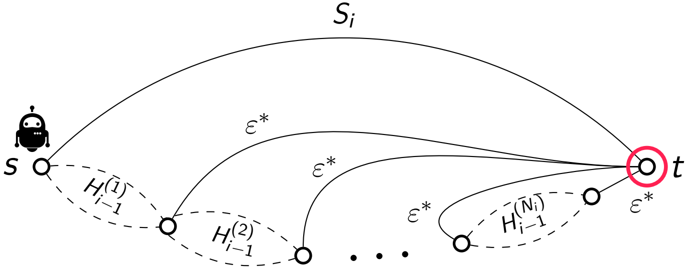
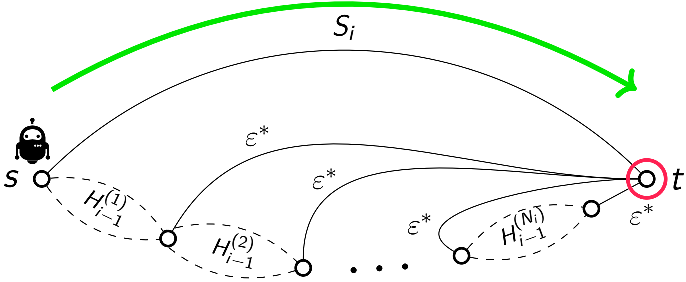


The kinds of strategies on H_i, and how to block them



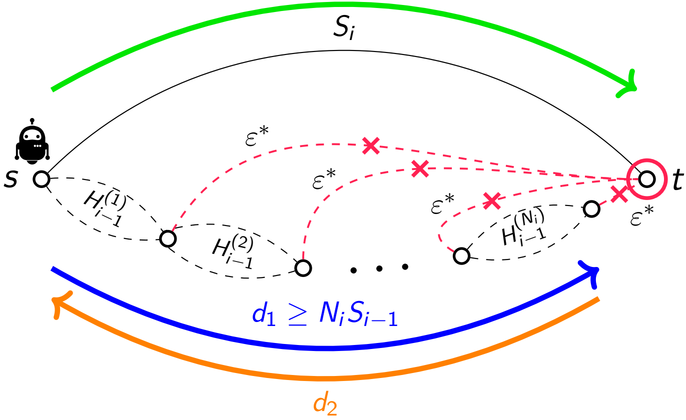
Reminder: We want ratio at least $i+1-\varepsilon$.
We have $N_i = i(i+1)$ and $S_i = (i+1)!$.

The kinds of strategies on H_i, and how to block them



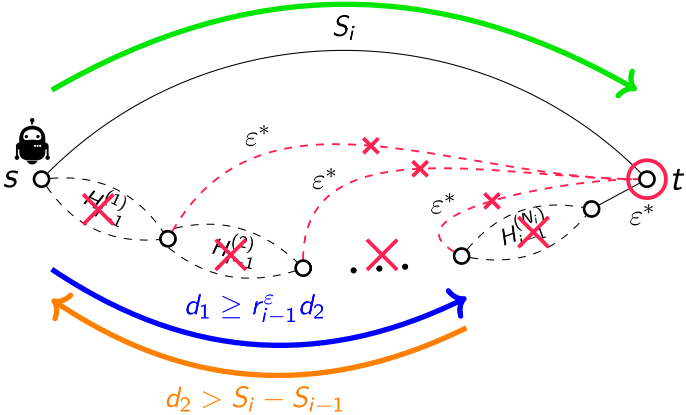
First strategy: Going through $st \Rightarrow$
Opponent blocks nothing, ratio is $\frac{S_i}{1+2\varepsilon^*}$.

The kinds of strategies on H_i, and how to block them



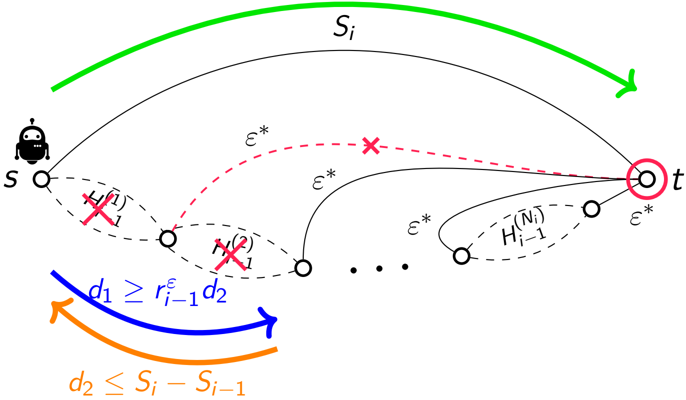
Second strategy: Trying to reach t by going below $\Rightarrow$ Opponent blocks all ε^*-edges to t, ratio is $\geq \frac{d_1+d_2+S_i}{S_i} > \frac{d_1+S_i}{S_i} \geq \frac{N_i S_{i-1}+S_i}{S_i} = \frac{i(i+1)i!}{(i+1)!} + 1 = i + 1.$

The kinds of strategies on H_i, and how to block them



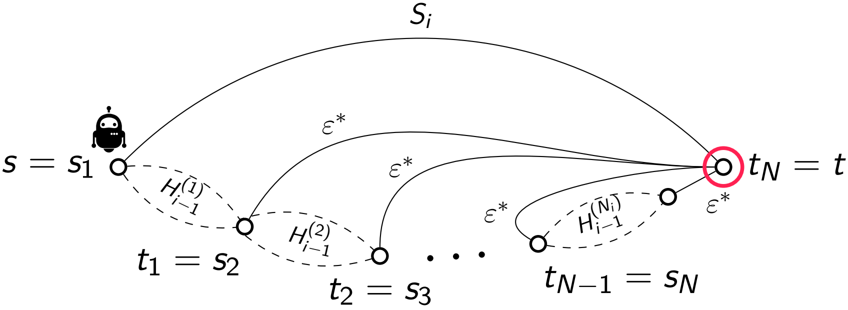
Third strategy: Exploring deep below $\Rightarrow$ Opponent makes traversing each copy of H_{i-1} costly, ratio is $\geq \frac{(1+r^{\varepsilon}_{i-1})d_2+S_i}{S_i} \geq \frac{(1+r^{\varepsilon}_{i-1})(S_i-S_{i-1})+S_i}{S_i} = \frac{(2+r^{\varepsilon}_{i-1})S_i-(1+r^{\varepsilon}_{i-1})S_{i-1}}{S_i} \geq 2 + r^{\varepsilon}_{i-1} - \frac{(1+i)i!}{(i+1)!} \geq r^{\varepsilon}_{i-1} + 1.$

The kinds of strategies on H_i, and how to block them



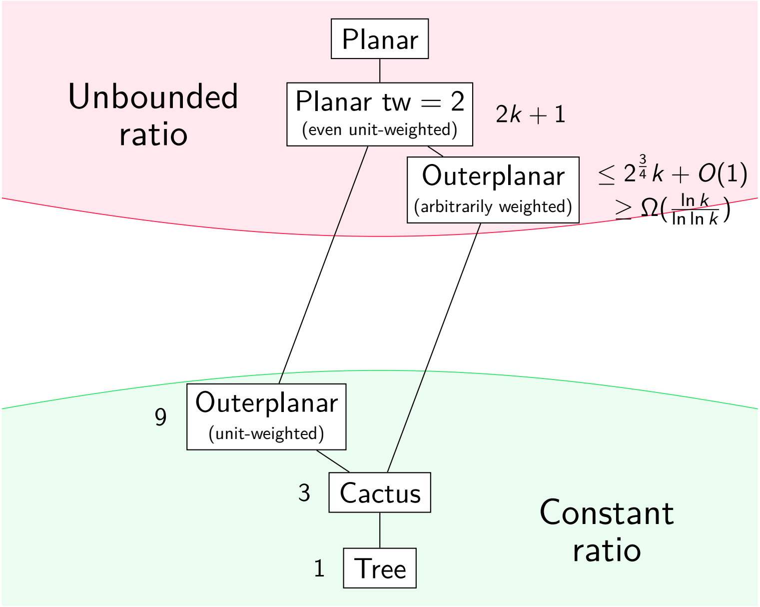
Fourth strategy: Exploring a little below $\Rightarrow$ Opponent makes traversing the first copies of H_{i-1} costly, but leaves the next ε^*-chord to t open, ratio is $\geq \frac{(1+r_{i-1}^{\varepsilon})d_2+S_i}{d_2+S_{i-1}+\varepsilon^*}$. Computations are technical, but end with ratio $\geq\ r_{i-1}^{\varepsilon} + 1$.

Finishing the proof: the competitive ratio is unbounded



- On the family H_i, the competitive ratio is always $\geq i + 1 + \varepsilon$ by blocking $\leq ((i+1)!)^2$ edges.
- We allow k blocked edges. By choosing a good value for i depending on k, we can have the competitive ratio greater than $g(k) = e^{W(\frac{\ln k}{2})} - 1$.
- Asymptotically, $\frac{\ln k}{\ln \ln k} \leq g(k) \leq \ln k$.

Summary



Planar
Unbounded ratio
Planar tw = 2
(even unit-weighted)
$2k+1$
Outerplanar
(arbitrarily weighted)
$\leq 2^{\frac{3}{4}}k + O(1)$
$\geq \Omega(\frac{\ln k}{\ln \ln k})$
9
Outerplanar
(unit-weighted)
3
Cactus
Constant ratio
1
Tree

Summary and perspectives!