Algorithms and hardness for Metric Dimension on directed graphs

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LIMOS, Clermont-Ferrand
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June 30, 2023

Where does it come from?

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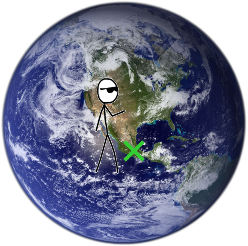

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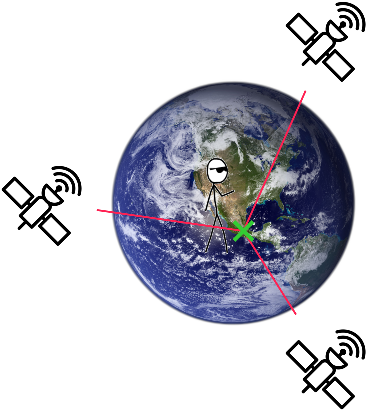

GPS, GLONASS, Galileo, Beidou, IRNSS, QZSS: use of at least four satellites

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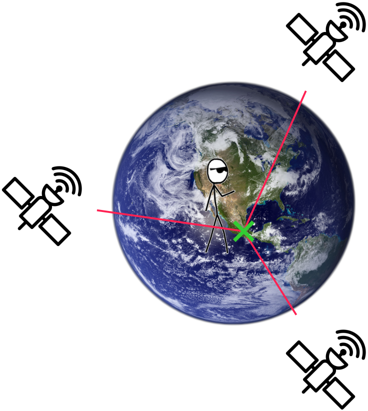

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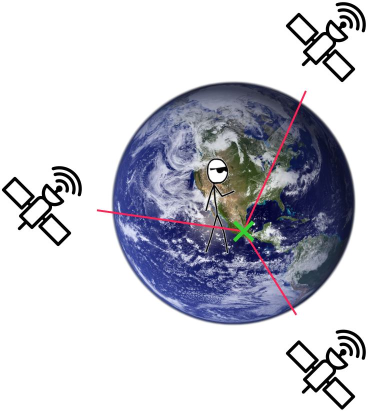

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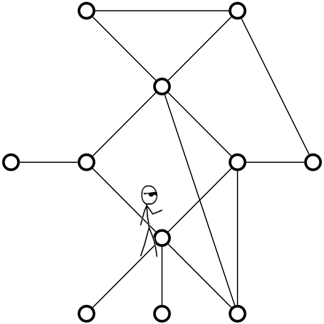

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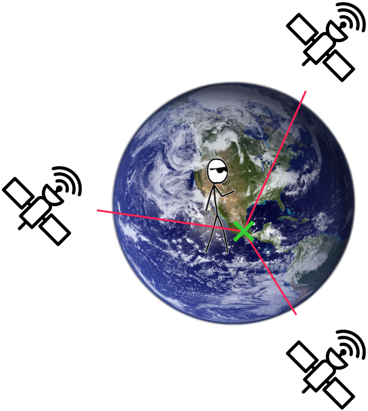

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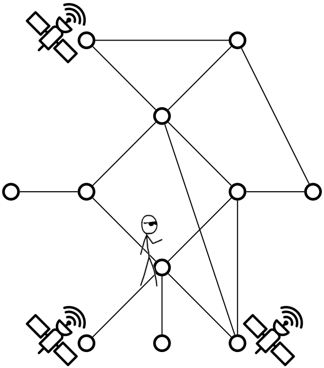

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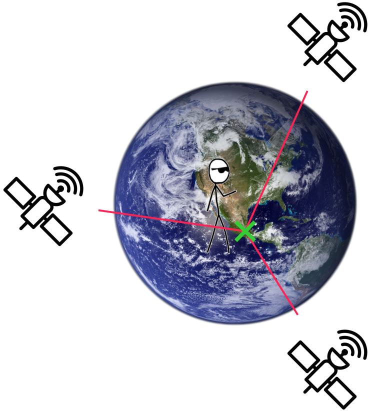

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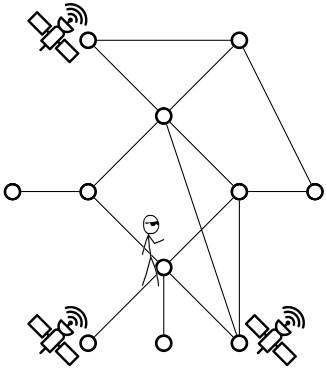

How many "satellites" would I need in a given graph?

Metric Dimension

Definition

b resolves u and v if $\mathrm{dist}(b,u) \neq \mathrm{dist}(b,v)$

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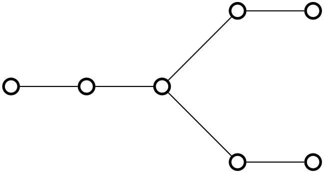

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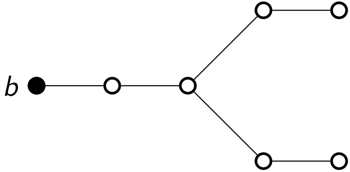

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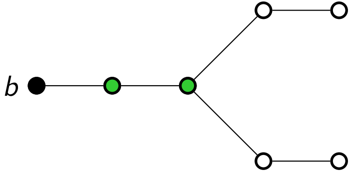

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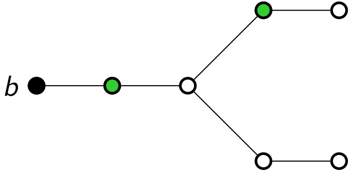

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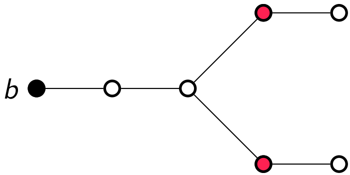

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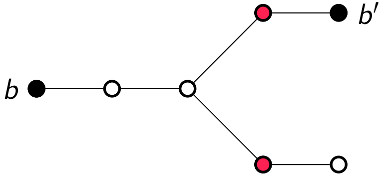

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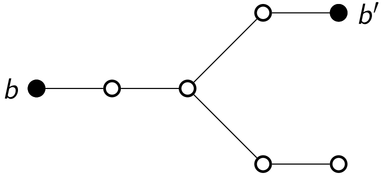

Resolving Set [Slater, 1975] [Harary & Melter, 1976]

$R \subseteq V(G)$ is a resolving set of G iff for every pair $\{u,v\}$, there is $b \in R$ that resolves u and v

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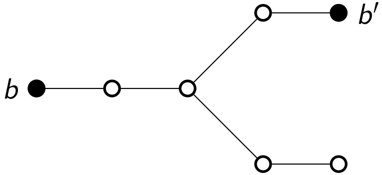

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$\mathrm{MD}(G) =$ minimum size of a resolving set of G

Basic results

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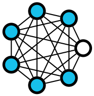

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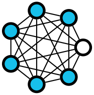

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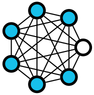

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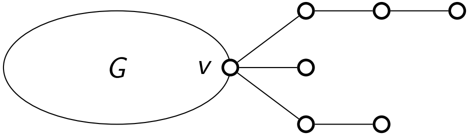
Legs

Paths with degree 2 inner vertices, and degree 1 and ≥ 3 end-points.



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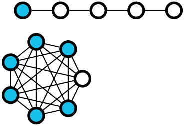

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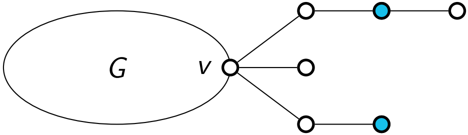
Paths with degree 2 inner vertices, and degree 1 and ≥ 3 end-points. If v has k legs, $k-1$ have ≥ 1 vertex in a resolving set.



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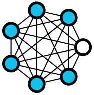

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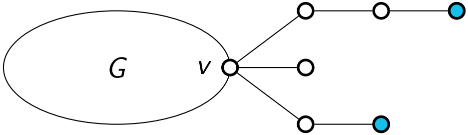

Simple leg rule: If v has $k \geq 2$ legs, select $k-1$ leg endpoints.

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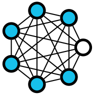

2. $\mathrm{MD}(G) = n-1 \Leftrightarrow G$ is K_n
3. Trees? The simple leg rule gives an optimal resolving set [Slater, 1975]

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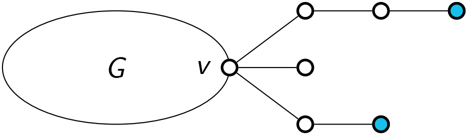
Paths with degree 2 inner vertices, and degree 1 and ≥ 3 endpoints. If v has k legs, $k-1$ have ≥ 1 vertex in a resolving set.



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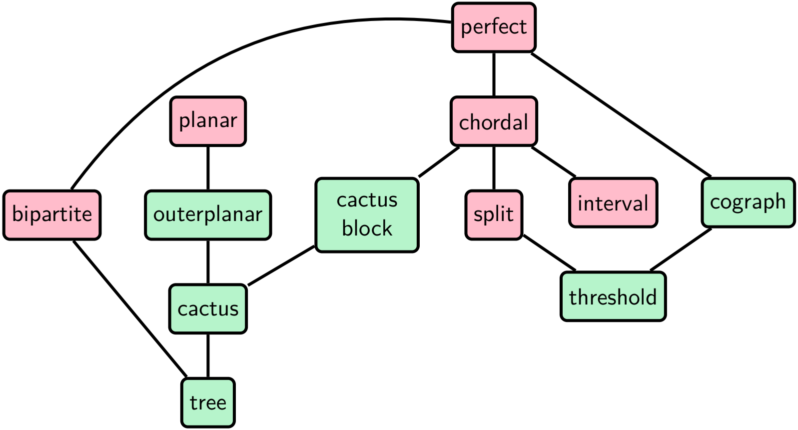

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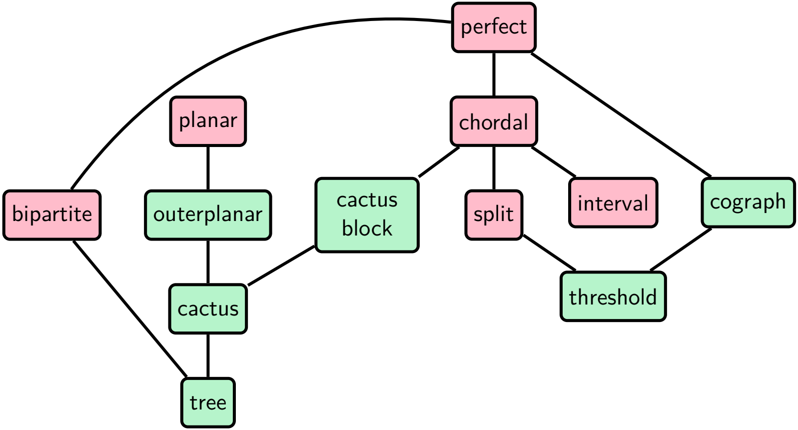

W[2]-complete and no better than $\log(n)$ approx in poly-time on subcubic graphs [HN13]

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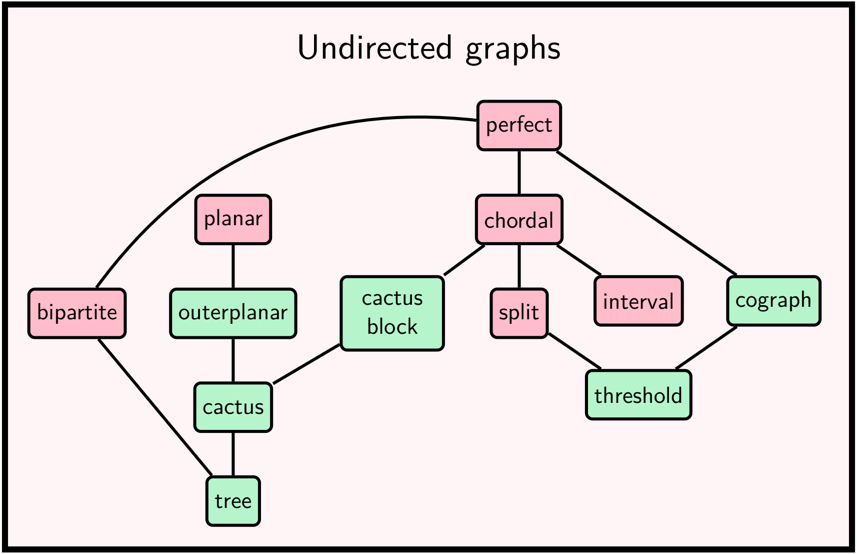

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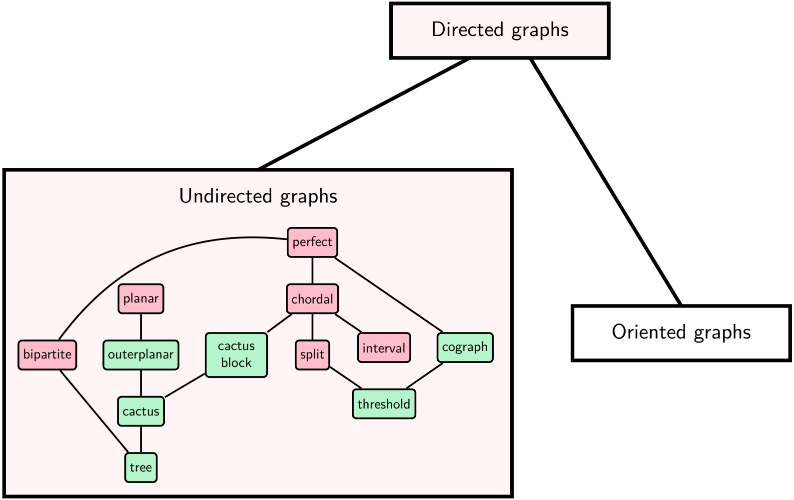

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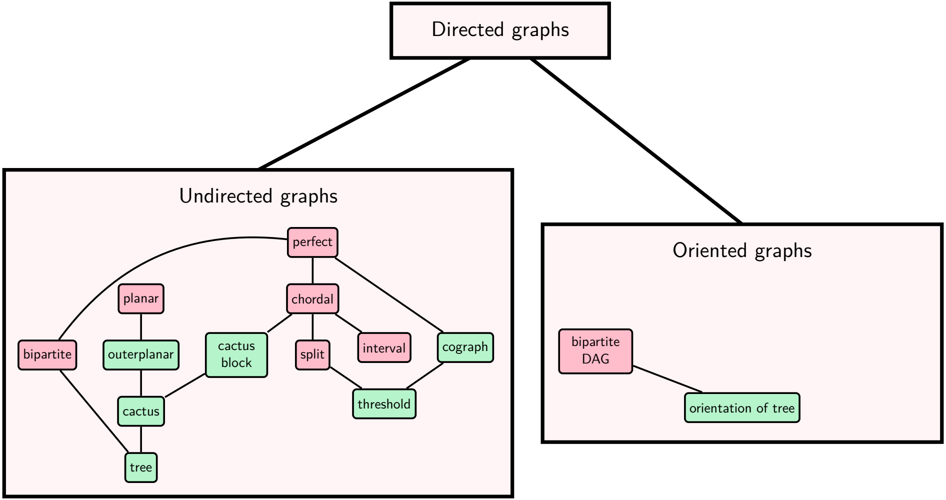

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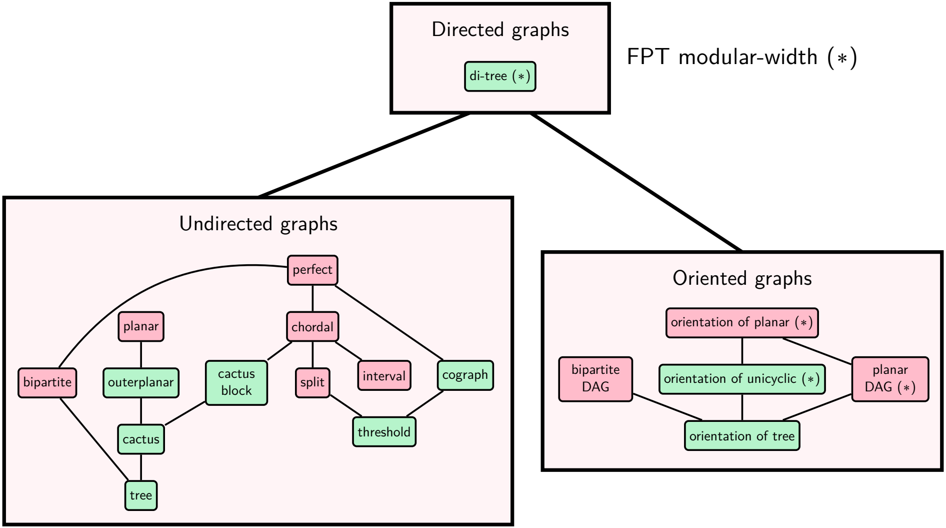
A difficult problem $(*)$ = our results



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Our results

Theorem [D., Foucaud & Hakanen, 2023]

Linear-time algorithms for minimum-size resolving sets of di-trees and orientations of unicyclic graphs.

Theorem [D., Foucaud & Hakanen, 2023]

Metric Dimension is NP-complete for planar triangle-free DAGs of maximum degree 6.

Theorem [D., Foucaud & Hakanen, 2023]

FPT algorithm for Metric Dimension parameterized by directed modular width.

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From a tree, transform every edge into an arc or a 2-cycle.

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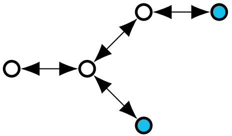
- Simple leg rule in strongly connected components



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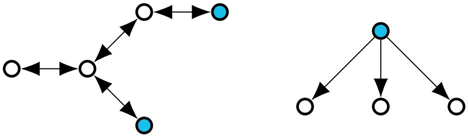

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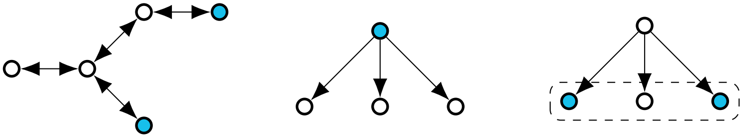
- Resolving sets of in-twins



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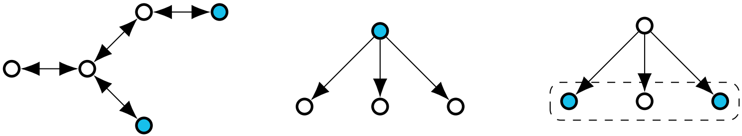


... but some refinement is needed!

Refining in-twins

Almost-in-twins

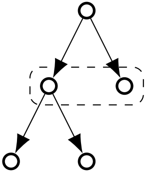
Vertices that share the same in-neighbour and:

Refining in-twins

Almost-in-twins

Vertices that share the same in-neighbour and:

- either are not in a nontrivial strongly connected component;



Refining in-twins

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Vertices that share the same in-neighbour and:

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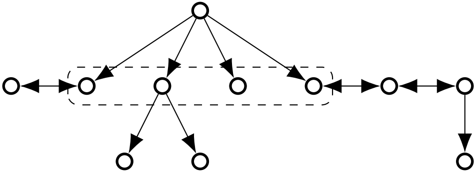
- or are the endpoint of a so-called *escalator*.



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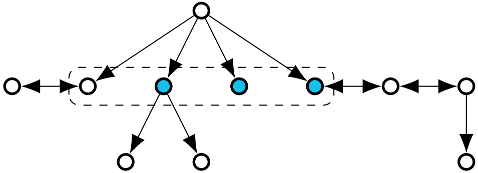


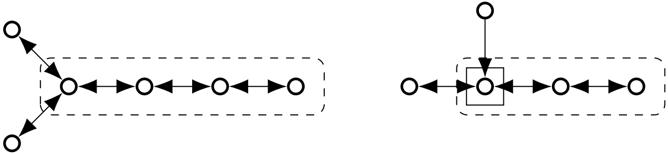
$\Rightarrow$ For each set of k almost-in-twins, take $k-1$ in the resolving set

Refining legs

Definition

In a strongly connected component, a special leg is a leg that:

- spans from a degree ≥ 3 (in the component) vertex or a vertex with an in-arc coming from outside the component



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In a strongly connected component, a special leg is a leg that:

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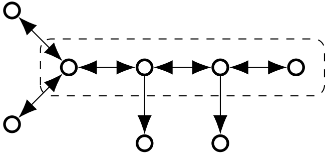

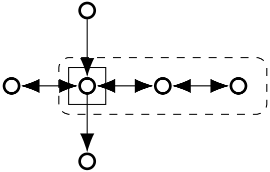

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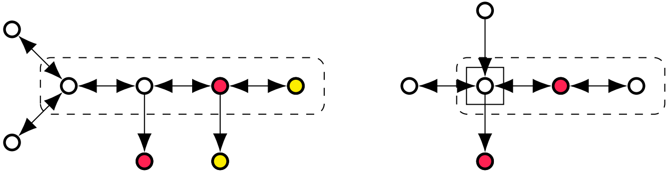

→ Conflict between pairs!

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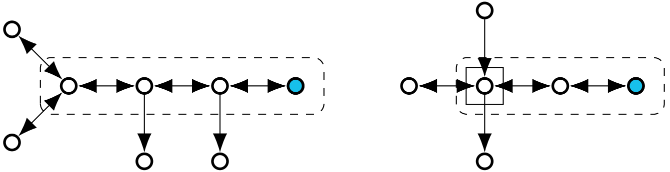

→ Conflict between pairs!

⇒ Take the endpoint of each special leg

The algorithm for di-trees

Theorem [D., Foucaud & Hakanen, 2023]

There is a linear-time algorithm computing a minimum-size resolving set of a di-tree.

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   2.1 Solve some special cases

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This gives a resolving set… which we prove is minimum-size!

Parameterized complexity

Theorem [D., Foucaud & Hakanen, 2023]

There is an $\mathcal{O}(n^3+m)+\mathcal{O}(t^5 2^{t^2} n)$ algorithm computing the metric dimension of a digraph of order n, size m and directed modular width at most t.

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1. Compute all the distances [Floyd-Warshall]
2. Obtain an optimal modular decomposition [McConnell & de Montgolfier, 2005]

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Generalized from [Belmonte *et al.*, 2017]

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2. Obtain an optimal modular decomposition [McConnell & de Montgolfier, 2005]
3. Start from the trivial modules, and combine them (dynamic programming)

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Definition [Gallai, 1967] (and many others)

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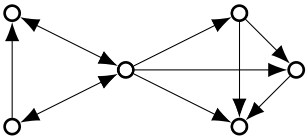

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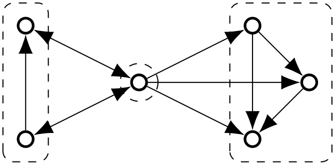

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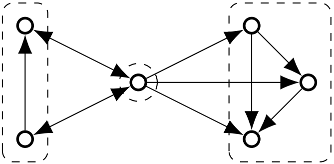

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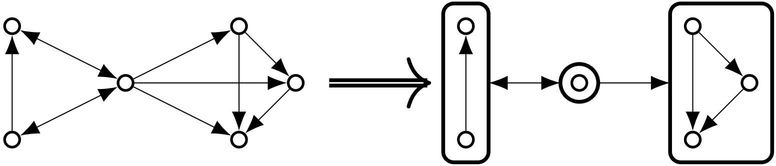

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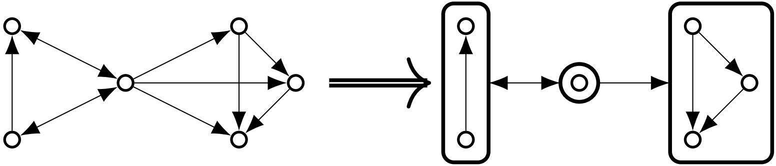

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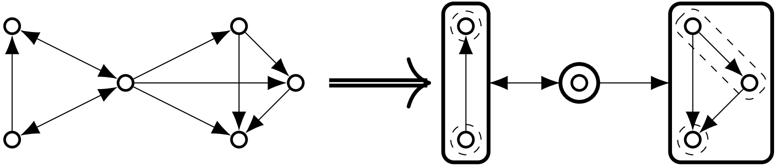

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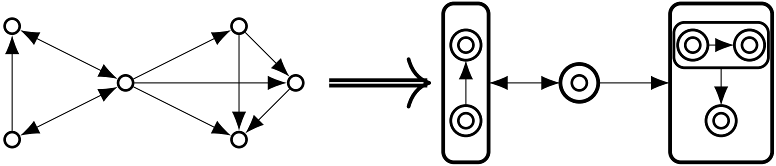

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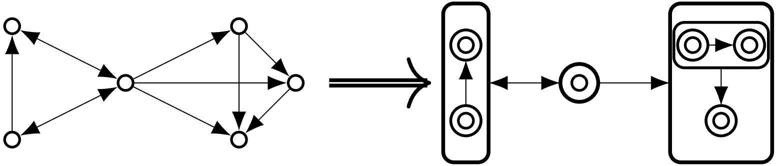

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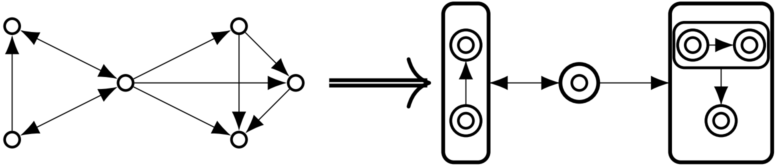

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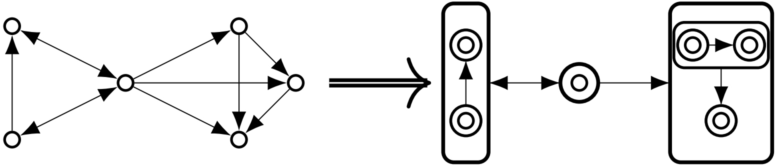

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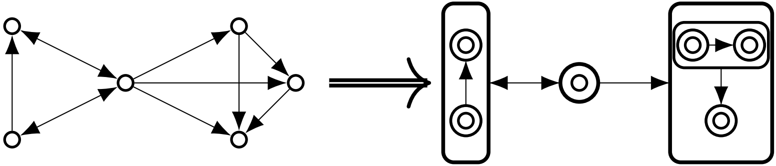

mw 3 width 3

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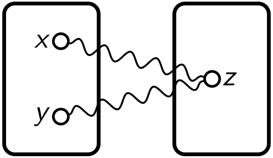

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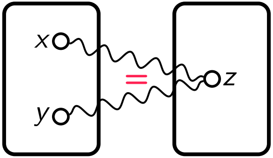

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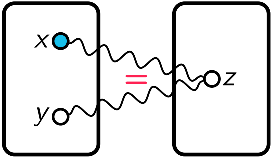

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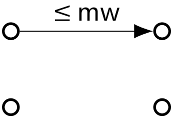

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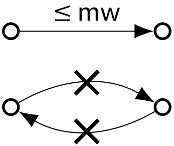

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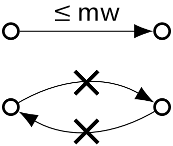

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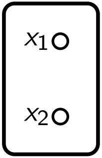

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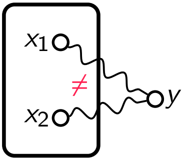

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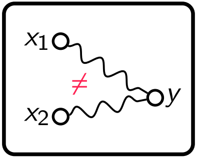

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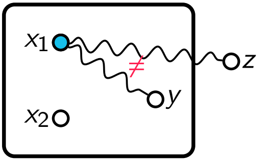

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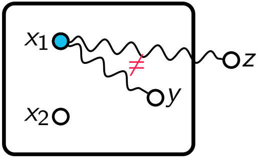

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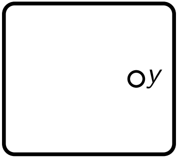

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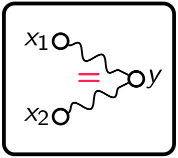

But what if, for some $y \in M_i$,
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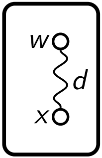

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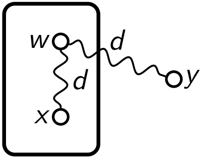

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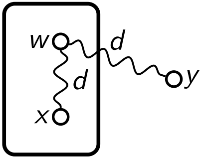

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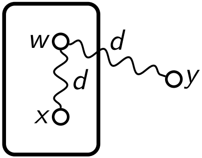

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... but $d \in \{1, \dots, \text{mw}, \infty\}$ so their number is **bounded by** mw+1 for each factor!

$\Rightarrow$ We can brute-force them when combining local solutions.

Final words

Our contribution to Metric Dimension on directed graphs

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- Linear-time algorithms (di-trees, orientations of unicyclic)
- FPT algorithm using modular decomposition

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